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DIFFEOMORPHISM COHOMOLOGY
IN BELTRAMI PARAMETRIZATION II : THE 1-FORMS

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Abstract

We study the 1-form diffeomorphism cohomologies within a local conformal Lagrangian Field Theory model built on a two dimensional Riemann surface with no boundary. We consider the case of scalar matter fields and the complex structure is parametrized by Beltrami differential. The analysis is first performed at the Classical level, and then we improve the quantum extension, introducing the current in the Lagrangian dynamics, coupled to external source fields. We show that the anomalies which spoil the current conservations take origin from the holomorphy region of the external fields, and only the differential spin 1 and 2 currents (as well their c.c) could be anomalous.

To be published in Journ. Math. Phys.

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1 Introduction

The most transparent and useful formulation for a field Theory surely is the one in which the locality is manifest.

The interest for this approach has been raised from the physical relevance of the local symmetries, such as gauge invariance for particle Physics and the diffeomorphism role in String and Gravitational models.

Now it appears in pratice that, for system of physical interest in which a local symmetry is realized, pointing out locality is a good appeling to investigate the deep meaning of the symmetry through the study of those local objects which, due to their invariance, have global properties.

This is the reason why it has been necessary to introduce in the literature the so called descent equations , and, plunged in the B.R.S. approach, they gave outstanding results in hunting anomalies, vertex operators, and so on.

In this scheme it has been necessary to provide the local objects as a form graduation, in relation to their invariance.

In a recent paper [1] we have used this strategy to study the anomalies [2] and the vertex operators in two dimensional diff-invariant models in Beltrami parametrization [3, 4, 8, 5, 11, 12], seen as local density quantities with form graduation equal to two, since they are to be integrated in two dimensional manifolds.

Indeed Beltrami parametrization is a good chance for studying the chiral resolution in conformal blocks [6] and holomorpic factorization [7, 3, 4, 5, 8], since the complex structure parametrization is realized in an automatic way.

In this paper we want to investigate the objects with form graduation equal to one, that is currents, which play an essential role in the symmetry realization and in the understanding of the links between the Lagrangian and the Hamiltonian approaches. In [9, 10] this analysis has been used in the Hamiltonian spirit, so its improvement within a Lagrangian framework is required.

Indeed the C^∞ smooth holomorphic functions

$$\left(z, \bar{z}\right) \longrightarrow \left(Z(z, \bar{z}), \bar{Z}(z, \bar{z})\right) \tag{1.1}$$

will individuate a set $(Z, \bar{Z})$ of coordinates and $(z, \bar{z})$ will be a reference frame of holomorphic coordinates on a Riemann surface.

Under this action, the set $(Z, \bar{Z})$ can be transformed under a reparametrization of $(z, \bar{z})$; so the invariance under the diffeomorphism:

$$\left(Z(z, \bar{z}), \bar{Z}(z, \bar{z})\right) \longrightarrow \left(Z'(z, \bar{z}), \bar{Z}'(z, \bar{z})\right) \tag{1.2}$$

can individuate models of relevant physical interest.

It is well known that infinitesimal action of (1.2) realizes a nilpotent operator s :

$$
\begin{aligned}
sZ &= (c^z\partial + c^{\bar z}\bar\partial)Z \\
sc^z &= (c^z\partial + c^{\bar z}\bar\partial)c^z \\
s^2 &= 0 \ .
\end{aligned}
\tag{1.3}
$$

where the ghost fields $c^z(z,\bar z), c^{\bar z}(z,\bar z)$ carry a $Q_{\Phi\Pi}$ charge equal to one.

So the diffeomorphism will change the $(Z,\bar Z)$ coordinates by means the action on the $(z,\bar z)$ ones. The Beltrami parametrization:

$$
dZ = \lambda(dz + \mu d\bar z)
$$
$$
\lambda = \partial Z
$$
$$
\mu = \frac{\bar\partial Z}{\partial Z}
$$

$$
\tag{1.4}
$$

with the compatibility condition:

$$
(\bar\partial - \mu\partial)\lambda = \lambda\partial\mu
\tag{1.5}
$$

is particularly attractive since in this approach conformal rescaling is seen as a diffeomorphism $(Z,\bar Z) \longrightarrow (Z',\bar Z')$ of the surface into itself with the same μ. Indeed when the Beltrami parameters are taken as constant the reparametrization operation is represented by complex analytic transition functions. Furthermore equivalence class of analytic atlases will identify a complex structure, and a conformal classe of 2-dimensional metrics. That is identifying conformal invariant models on a Riemaniann manifold needs a carefull description to factorize this equivalence arbitrariness. However, not all diffeomorphisms of the surface into itself amount to just a conformal rescaling. The intrinsic geometry of the surface is determined by the metric tensor (which is a coordinate-free object), and other changes in the metric produce confomally inequivalent surfaces.

Furthermore in a Lagrangian Field Theory model the Beltrami differentials are the appropriate sources of the energy momentum tensor components, whose short distance products will define the algebraic construction [6] of conformal current algebra.

Our purpose is here to characterize in a cohomological way all the diffeomorphism conserved currents, first at a Classical level an then to extend their properties (first of all their conservation) to the Quantum one.

Being the Action an invariant $(1,1)$ tensor, we shall suppose that in a conformal invariant theory the matter fields are realized on the Riemaniann manifold by local tensor fields $\Phi_{j\bar j}(Z,\bar Z)dZ^j d\bar Z^{\bar j}$ of weight $(j,\bar j)$ invariant under change of holomorphic charts

$$
\Phi_{j\bar j}(Z,\bar Z)^\alpha dZ_\alpha^j d\bar Z_\alpha^{\bar j} = \Phi_{j\bar j}(Z,\bar Z)^\beta dZ_\beta^j d\bar Z_\beta^{\bar j}
\tag{1.6}
$$

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It is possible also to define, via the diffeomorphism action restricted to dilatations, the geometric dimensions defined as:

$$\text{Dim} = N(\downarrow) - N(\uparrow) + Q_{\Phi\Pi} . \tag{1.7}$$

The B.R.S. realization of the infinitesimal diff-variations then reads:

$$
\begin{aligned}
sZ &= \gamma^Z \equiv \lambda_z^Z(c^z + \mu_{\bar{z}}^z c^{\bar{z}}) \\
s\Phi_{j\bar{j}}(Z,\bar{Z}) &= (\gamma^Z \partial_Z + \gamma^{\bar{Z}} \partial_{\bar{Z}})\Phi_{j\bar{j}}(Z,\bar{Z}) \\
s\gamma^Z &= s\gamma^{\bar{Z}} = 0 \\
s^2 &= 0 .
\end{aligned}
\tag{1.8}
$$

and its complex conjugates.

Going to little "z" indices it writes:

$$
\begin{aligned}
sZ &= \lambda(c + \mu\bar{c}) \equiv \gamma \\
s\lambda &= (c\cdot\partial)\lambda + \lambda(\partial c + \mu\partial\bar{c}) \equiv \partial(\lambda(c + \mu\bar{c})) \\
s\mu &= (c\cdot\partial)\mu - \mu(\partial c + \mu\partial\bar{c}) + \bar{\partial}c + \mu\bar{\partial}\bar{c} \\
sc &= (c\cdot\partial)c \\
s\phi_{j,\bar{j}} &= (c\cdot\partial)\phi_{j,\bar{j}} + \Big(j(\partial c + \mu\partial\bar{c}) + \bar{j}(\bar{\partial}\bar{c} + \bar{\mu}\bar{\partial}c)\Big)\phi_{j,\bar{j}} ,
\end{aligned}
\tag{1.9}
$$

with of course the complex conjugate expressions.

The matter fields are parametrized as:

$$
\Phi_{j\bar{j}}(Z,\bar{Z})dZ^j d\bar{Z}^{\bar{j}} = \phi_{j\bar{j}}(z,\bar{z})(dz + \mu_{\bar{z}}^z(z,\bar{z})d\bar{z})^j(d\bar{z} + \bar{\mu}(z,\bar{z})dz)^{\bar{j}}
$$

$$\tag{1.10}$$
$$\tag{1.11}$$

with:

$$
\Phi_{j\bar{j}}(Z,\bar{Z}) = \frac{\phi_{j\bar{j}}(z,\bar{z})}{\lambda^j(z,\bar{z})\bar{\lambda}^{\bar{j}}(z,\bar{z})}.
$$

$$\tag{1.12}$$

The previous variations (1.9) will define a B.R.S. local operator δ such that $\delta^2 = 0$ acting on the space of the previous fields and their derivatives considered as independent monomials coordinates as in Ref [14]

So, even if the $(Z,\bar{Z})$ frame will describe the model, the use of little "z" coordinates is particularly useful (as remarked in [1, 15], since the the derivative operator can be defined, in the above mentioned scheme, by means of the δ operator and the "little" c ghosts as we shall see in the following).

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Let $Q_\star$ be the physical charges in each tensorial sectors, such that the $\star$ label will sum up covariant and controvariant "big" indices. These charges will derive from currents as:

$$Q_\star = \int \left(\mathcal{J}_{Z,\star} dZ + \mathcal{J}_{\bar{Z},\star} d\bar{Z} \right) = \int \mathcal{J}^0_{1,\star}(Z, \bar{Z}) \tag{1.13}$$

so that the form degree is with respect the "big" indices.

The diffeomorphism invariance will assure that it will be a counterpart in the "little" indices:

$$Q_\star = \int \left(\mathcal{J}_{z,\star} dz + \mathcal{J}_{\bar{z},\star} d\bar{z} \right) = \int \mathcal{J}^0_{1,\star}(z, \bar{z}) \tag{1.14}$$

The aim of this paper is to study the diff-invariant charges, that is the quantities $Q_\star$ which verify:

$$\delta Q_\star = 0. \tag{1.15}$$

It is well known that such a symmetry has to require, at least at the quantum level, conserved currents, which is a "local" constraint well defined with respect to a reference frame; but the diff-invariance (1.1) puts on the same footing a large class of systems of coordinates: so it may be interesting to investigate how the symmetry, realized in a local way, will generalize at each chart the currents conservation. We shall find that the two dimensional character of the theory, if it is defined on manifolds without boundary, requires, for the existence of the charge, the holomorphic factorization of currents in the $Z(z, \bar{z})$ (or its c.c) variable. This fact will have many important consequences that we shall investigate in this paper. First of all the fact that all the local currents will have definite covariance properties, that is their will be $(n+1, 0)$ (or their c.c) true tensors.

Furthermore our aim will be to extend at the Quantum level all the properties found, established at the classical one. For this reason we have to put the currents inside an invariant Action by coupling them to external fields, and we have to study the perturbative renormalization of the model. We shall show that only the spin 1 and 2 currents will spoil conservation at the quantum level, while all the other ones will maintain all the classical symmetries. The paper is organized as follows:

In Section 2 we shall solve the 1-forms descent equation deriving from the diff-mod **d** invariance and in particular we shall show that diff-current conservations can be derived from the diff-cohomology, so they cannot depend on the particular choice of coordinates: we shall establish these relations in the "true" $(Z, \bar{Z})$ coordinates as well as in the reference frame $(z, \bar{z})$. We shall show that the charge existence condition on a Riemann surface with no boundary, implies holomorphic constraints for the currents.

In Section 3 we shall briefly recover the previous results in a Lagrangian two dimensional dynamics, which forces a two-forms analysis. This artillery allows a perturbative quantum extension of the diff-invariance in order to find the possible obstructions to both the current covariance and the current conservation.

An Appendix is devoted to some computational details using the spectral sequences method [14][15]

2 The 1-forms in the δ-cohomology

In this section we want to analyze the descent equation of 1-forms, already done in [9][10], but following the spirit of [1].

To be more accurate we shall first relate the diff-mod **d** cohomology to the local unintegrated functions, by solving the ordinary differential action in term of the B.R.S. operator for diffeomorphism.

Furthermore we shall show that all the uncharged elements which are solutions of the 1-forms descent equations will indentify conserved currents: more exactly, the diffeomorphism cohomology alone will specialize current conservation both on the "little" and "big" coordinates:

This fact has an important consequence for the two-dimensional character of the theory: the current conservation condition will admit inversion formula, and, on a manifold without boundary, the holomorphic factorization of currents will be obtained.

We stress that this is, in our framework, a classical level analysis which support a particular importance for proving the stability properties of theory; the quantum extension will need more accurancy.

2.1 The consistency equations and the current conservation

We shall start from those objects (defined in the general reference frame $(z, \bar{z})$)

$$Q_\star = \int \left(\mathcal{J}_{z,\star} dz + \mathcal{J}_{\bar{z},\star} d\bar{z} \right) = \int \mathcal{J}_{1,\star}^0(z, \bar{z}) \tag{2.1}$$

(and the form degree is relative to the "little" indices)

which are elements of the δ cohomology:

$$\delta Q_\star = 0; Q_\star \neq \delta \hat{Q}_\star \tag{2.2}$$

In terms of local quantities the cocycle equation will imply:

$$\delta \mathcal{J}_{1,\star}^0(z, \bar{z}) + d \mathcal{J}_{0,\star}^1(z, \bar{z}) = 0 \tag{2.3}$$

where δ operator acts in the space of local unintegrated functions as described in (1.9); its full complete description will be found in the Appendix(A.3).

The previous equation will characterize $\mathcal{J}_{1,\star}^0(z, \bar{z})$ as an element of the diff-mod **d** cohomology .

From (2.3) we derive the expressions of the well-known descent equations:

$$\delta \mathcal{J}_{1,\star}^0(z, \bar{z}) + d \mathcal{J}_{0,\star}^1(z, \bar{z}) = 0$$
$$\delta \mathcal{J}_{0,\star}^1(z, \bar{z}) = 0 \tag{2.4}$$

so that the bottom current writes

$$\mathcal{J}_{0,\star}^1(z, \bar{z}) = \mathcal{J}_{0,\star}^{\natural 1}(z, \bar{z}) + \delta \widehat{\mathcal{J}}_{0,\star}^0(z, \bar{z}) \tag{2.5}$$

where $\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})$ is an element of the cohomology of δ in the space of the unintegrated functions.

Writing the differential operator in term of the δ as in [15, 1]

$$
\partial = \left\{ \delta, \frac{D}{Dc(z,\bar{z})} \right\}
$$
$$
\bar{\partial} = \left\{ \delta, \frac{D}{D\bar{c}(z,\bar{z})} \right\}
$$

(2.6)

(we remark that it is only true in the "little" coordinates reference frame) one gets by a direct substitution in eqs.(2.4)

$$
\delta \mathcal{J}^0_{1,\star}(z,\bar{z}) + \left(dz\left\{ \delta, \frac{D}{Dc(z,\bar{z})} \right\} + d\bar{z}\left\{ \delta, \frac{D}{D\bar{c}(z,\bar{z})} \right\} \right) (\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z}) + \delta\widehat{\mathcal{J}}^0_{0,\star}(z,\bar{z})) =
$$

$$
= \delta \left[\mathcal{J}^0_{1,\star}(z,\bar{z}) + \left(dz\frac{D}{Dc(z,\bar{z})} + d\bar{z}\frac{D}{D\bar{c}(z,\bar{z})} \right) \mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z}) + d\widehat{\mathcal{J}}^0_{0,\star}(z,\bar{z}) \right] = 0
$$

which is solved by the 1-form

$$
\mathcal{J}^0_{1,\star}(z,\bar{z}) = \mathcal{J}^{\natural 0}_{1,\star}(z,\bar{z}) - \left(dz\frac{D}{Dc(z,\bar{z})} + d\bar{z}\frac{D}{D\bar{c}(z,\bar{z})} \right) \mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z}) - d\widehat{\mathcal{J}}^0_{0,\star}(z,\bar{z}) + \delta\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z})
$$

(2.7)

This is the fundamental formula which relates the elements of the diff-mod **d** cohomology to the elements of of δ one.

It will be very useful to calculate this cohomological space: this will be done below. Let us however recall the most important result coming from this calculation and show their consequences.

First of all, denoting by $N_z(\downarrow)$ and $N_z(\uparrow)$ the lower and upper "little" indices counting operators respectively, we shall show in the Appendix that:

$$
(N_z(\downarrow) - N_z(\uparrow))\mathcal{J}^{\natural 0}_{1,\star}(z,\bar{z}) = (N_{\bar{z}}(\downarrow) - N_{\bar{z}}(\uparrow))\mathcal{J}^{\natural 0}_{1,\star}(z,\bar{z}) = 0
$$

(2.8)

which as 1-form implies:

$$
\mathcal{J}^{\natural 0}_{1,\star}(z,\bar{z}) \equiv 0
$$

(2.9)

so (2.7) reduces to:

$$
\mathcal{J}^0_{1,\star}(z,\bar{z}) = - \left(dz\frac{D}{Dc(z,\bar{z})} + d\bar{z}\frac{D}{D\bar{c}(z,\bar{z})} \right) \mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z}) - d\widehat{\mathcal{J}}^0_{0,\star}(z,\bar{z}) + \delta\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z})
$$

(2.10)

and the dimensions of the currents are given by the Z(big!) index content.

$$\text{Dim} = N_Z(\downarrow) - N_Z(\uparrow) \ . \tag{2.11}$$

where $N_Z(\downarrow)$ and $N_Z(\uparrow)$ are, as can be easily understood, the counting operators of the "big" lower and upper indices respectively.

Then (2.10) tell us that the current conservation is a direct consequence of the diffeomorphism invariance.

Indeed, as pointed out in [9] applying d on eq. (2.3), we get:

$$d\delta \mathcal{J}^0_{1,\star}(z,\bar z) = 0 \tag{2.12}$$

that is, since $\left[d,\delta\right] = 0$, in terms of the δ-cohomology functions, we get:

$$d\mathcal{J}^0_{1,\star}(z,\bar z) = \mathcal{J}^{0;\natural}_{2,\star}(z,\bar z) + \delta \widehat{\mathcal{J}^{-1}_{1,\star}}(z,\bar z) \tag{2.13}$$

we want here to show that the $\mathcal{J}^0_{2,\star}(z,\bar z)^\natural$ term of the r.h.s. is zero.

From the very definition we have:

$$d\mathcal{J}^0_{1,\star}(z,\bar z) = -\left(\bar\partial \frac{D}{Dc(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z) dz \wedge d\bar z + \partial \frac{D}{D\bar c(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z) d\bar z \wedge dz \right) + \delta d\widehat{\mathcal{J}^{-1}_{1,\star}}(z,\bar z)$$
$$= \left(-\bar\partial \frac{D}{Dc(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z) + \partial \frac{D}{D\bar c(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z) \right) d\bar z \wedge dz + \delta d\widehat{\mathcal{J}^{-1}_{1,\star}}(z,\bar z) \tag{2.14}$$

on the one hand we have from (2.6)

$$d\mathcal{J}^0_{1,\star}(z,\bar z) = \left(-\left\{\delta, \frac{D}{D\bar c(z,\bar z)}\right\} \frac{D}{Dc(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z) \right.$$
$$\left. +\left\{\delta, \frac{D}{Dc(z,\bar z)}\right\} \frac{D}{D\bar c(z,\bar z)} \mathcal{J}^{\natural 1}_{0,\star}(z,\bar z)\right) d\bar z \wedge dz + \delta d\widehat{\mathcal{J}^{-1}_{1,\star}}(z,\bar z) \tag{2.15}$$

On the other hand, by using directly, in (2.14):

$$\left[\partial, \frac{D}{D\bar c(z,\bar z)}\right] = \left[\bar\partial, \frac{D}{Dc(z,\bar z)}\right] = 0 \tag{2.16}$$

we get

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$$dJ^0_{1,\star}(z,\bar{z}) = -\left(\frac{D}{Dc(z,\bar{z})}\bar{\partial}\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})dz \wedge d\bar{z} + \frac{D}{D\bar{c}(z,\bar{z})}\partial\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})d\bar{z} \wedge dz\right)$$

$$+\delta d\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z}) - \left(\frac{D}{Dc(z,\bar{z})}\left\{\delta, \frac{D}{D\bar{c}(z,\bar{z})}\right\}\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})\right.$$

$$\left.+\frac{D}{D\bar{c}(z,\bar{z})}\left\{\delta, \frac{D}{Dc(z,\bar{z})}\right\}\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})\right) d\bar{z} \wedge dz + \delta d\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z})$$

$$\tag{2.17}$$

Comparison of eqs now (2.15) and (2.17):

$$dJ^0_{1,\star}(z,\bar{z}) = -\delta\left(\frac{D}{Dc(z,\bar{z})}\frac{D}{D\bar{c}(z,\bar{z})}\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})d\bar{z} \wedge dz - d\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z})\right) \tag{2.18}$$

that is, the $\mathcal{J}^0_{2,\star}(z,\bar{z})^\natural$ obstruction term, which a priori occurred in (2.13) does not appear.

Moreover the term $\frac{D}{Dc(z,\bar{z})}\frac{D}{D\bar{c}(z,\bar{z})}\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})d\bar{z} \wedge dz$ can contribute to the Φ, Π uncharged sector only if $\mathcal{J}^{\natural 1}_{0,\star}(z,\bar{z})$ will contain negative charged fields:so, since we remove the anti-ghosts fields, after imposing their equations of motion through the gauge fixing, we can assume the only Φ, Π charged negative fields are only those which occur in the Lagrangian coupled to the B.R.S. transformation.

We shall show in the Appendix that in the δ cohomology space no negative charged field can appear, hence:

$$dJ^0_{1,\star}(z,\bar{z}) = \delta d\widehat{\widehat{\mathcal{J}}}^{-1}_{1,\star}(z,\bar{z}) \tag{2.19}$$

but recalling that $\mathcal{J}^0_{1,\star}(z,\bar{z})$ is a representative of an equivalence class, and is defined modulo arbitrary δ contributions, the current conservation is realized only for elements $\mathcal{J}^0_{1,\star}(z,\bar{z}) - \delta\widehat{\widehat{\mathcal{J}}}^{-1}_{1,\star}(z,\bar{z})$ which will define "locally" the conserved current $\tilde{\mathcal{J}}^0_{1,\star}(z,\bar{z})$, such that:

$$d\tilde{\mathcal{J}}^0_{1,\star}(z,\bar{z}) \equiv d\left(\mathcal{J}^0_{1,\star}(z,\bar{z}) - \delta\widehat{\widehat{\mathcal{J}}}^{-1}_{1,\star}(z,\bar{z})\right) = 0 \tag{2.20}$$

2.2 The local δ cohomology

As pointed out in the formula (2.10), we have shown that the elements of the diff-mod **d** cohomology can be easily derived from the ones of the δ cohomology in the space of local functions. The aim of this part is to solve:

$$\delta\mathcal{J}^{\natural r}_{0,\star}(z,\bar{z}) = 0 \tag{2.21}$$

where the upper index r will label the Φ, Π charge and the lower index the form degree respectively. The Φ, Π charge sector we are interested in, is the one with $r = 1$

If we decompose the cohomology spaces into their underivated ghost content:

$$\mathcal{J}^{\natural r}_{0,\star}(z,\bar{z}) = \mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) + c\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \bar{c}\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) + c\bar{c}\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) \qquad (2.22)$$

where $\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}), \mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}), \mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}), \mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z})$ do not contain underivated ghosts, the condition (2.21) will imply the following system:

$$\widehat{\delta}\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \qquad (2.23)$$

$$\left(-\widehat{\delta} + \partial c\right)\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \partial\bar{c}\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) + \partial\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \qquad (2.24)$$

$$\left(-\widehat{\delta} + \bar{\partial}\bar{c}\right)\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) + \bar{\partial}c\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \bar{\partial}\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \qquad (2.25)$$

$$\left(\widehat{\delta} - \partial c - \bar{\partial}\bar{c}\right)\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) - \bar{\partial}\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \partial\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) = 0 \qquad (2.26)$$

where

$$\widehat{\delta} \equiv \delta - c\partial - \bar{c}\bar{\partial} \qquad (2.27)$$

hence:

$$\widehat{\delta}\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \qquad (2.28)$$

$$\left(-\widehat{\delta} + (\partial - \bar{\mu}\bar{\partial})c\right)(\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) - \bar{\mu}\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z})) + (\partial - \bar{\mu}\bar{\partial})\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \quad (2.29)$$

$$\left(-\widehat{\delta} + (\bar{\partial} - \mu\partial)\bar{c}\right)(\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) - \mu\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z})) + (\bar{\partial} - \mu\partial)\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = 0 \quad (2.30)$$

$$\left(\widehat{\delta} - \partial c - \bar{\partial}\bar{c}\right)\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) - \bar{\partial}\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \partial\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) = 0 \qquad (2.31)$$

In the Appendix we shall show that in a general Lagrangian model, in which, for the sake of simplicity the matter fields are taken to be scalar, and the only Φ, Π negative charged fields are the external ones coupled to

the B.R.S. variations (and the gauge terms are taken away by solving $\widehat{\delta}^2 = 0$) the $\widehat{\delta}$ cohomology space does not contain negative charged fields, so for $r \neq 0$ the solution of (2.28) is $\widehat{\delta}$ trivial:

$$\mathcal{J}^{\natural,0r}_{0,\star}(z,\bar{z}) = \widehat{\delta}\widetilde{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z}) \qquad (2.32)$$

Then, using:

$$\left[\widehat{\delta}, \partial - \bar{\mu}\bar{\partial}\right] = \left((\partial - \bar{\mu}\bar{\partial})c\right)(\partial - \bar{\mu}\bar{\partial}) \qquad (2.33)$$

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we get

$$\left(-\widehat{\delta} + (\partial - \bar{\mu}\bar{\partial})c\right)\left(\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) - \bar{\mu}\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) + (\partial - \bar{\mu}\bar{\partial})\widehat{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z})\right) = 0 \quad (2.34)$$

$$\left(-\widehat{\delta} + (\bar{\partial} - \mu\partial)\bar{c}\right)\left(\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) - \mu\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + (\bar{\partial} - \mu\partial)\widehat{\mathcal{J}}^{\bar{r-1}}_{0,\star}(z,\bar{z})\right) = 0 \quad (2.35)$$

So, if we define:

$$\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) - \bar{\mu}\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) + (\partial - \bar{\mu}\bar{\partial})\widehat{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z}) \equiv \lambda\mathcal{J}^{r-1}_{Z,0,\star}(Z,\bar{Z})(1 - \mu\bar{\mu}) \quad (2.36)$$

we get

$$\widehat{\delta}\mathcal{J}^{r-1}_{Z,0,\star}(Z,\bar{Z}) = 0 \quad (2.37)$$

which is solved, according to the results given in the Appendix, by:

$$\mathcal{J}^{r-1}_{Z,0,\star}(Z,\bar{Z}) = \mathcal{J}^{\natural,r-1}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z}) \quad (2.38)$$

where $\mathcal{J}^{\natural,r-1}_{Z,0,\star}(Z,\bar{Z})$ and $\bar{\mathcal{J}}^{\natural,r-1}_{\bar{Z},0,\star}(Z,\bar{Z})$ are elements of the $\widehat{\delta}$ cohomology space.

Next defining:

$$\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) = \partial\widehat{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z}) + \lambda(\mathcal{J}^{\natural,r-1}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z})) + \bar{\mu}\bar{\lambda}(\bar{\mathcal{J}}^{\natural,r-1}_{\bar{Z},0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\bar{\mathcal{J}}}^{r-2}_{\bar{Z},0,\star}(Z,\bar{Z})) \quad (2.39)$$

$$\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z}) = \bar{\partial}\widehat{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z}) + \bar{\lambda}(\bar{\mathcal{J}}^{\natural,r-1}_{\bar{Z},0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\bar{\mathcal{J}}}^{r-2}_{\bar{Z},0,\star}(Z,\bar{Z})) + \mu\lambda(\mathcal{J}^{\natural,r-1}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z})) \quad (2.40)$$

we have

$$\left(\widehat{\delta} - \partial c - \bar{\partial}\bar{c}\right)\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) - \bar{\partial}\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) + \partial\mathcal{J}^{\natural r-1}_{0\bar{z},\star}(z,\bar{z})$$

$$\equiv \left(\widehat{\delta} - \partial c - \bar{\partial}\bar{c}\right)\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) - \lambda\bar{\lambda}(1 - \mu\bar{\mu})\left(\partial_{\bar{Z}}(\mathcal{J}^{\natural,r-1}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z}))\right.$$

$$\left.-\partial_Z(\bar{\mathcal{J}}^{\natural,r-1}_{\bar{Z},0,\star}(Z,\bar{Z}) + \widehat{\delta}\widehat{\bar{\mathcal{J}}}^{r-2}_{\bar{Z},0,\star}(Z,\bar{Z}))\right) = 0$$

$$(2.41)$$

By introducing:

$$\mathcal{J}^{\natural r-2}_{0z\bar{z},\star}(z,\bar{z}) = \lambda\bar{\lambda}(1 - \mu\bar{\mu})\Lambda^{r-2}_{Z,\bar{Z},\star}(Z,\bar{Z}) \quad (2.42)$$

we obtain:

$$\widehat{\delta}\left(\Lambda_{Z,\bar{Z},\star}^{r-2}(z,\bar{z})-\partial_{\bar{Z}}\widehat{\bar{\mathcal{J}}}_{\bar{Z},0,\star}^{r-2}(Z,\bar{Z})+\partial_{\bar{Z}}\widehat{\mathcal{J}}_{Z,0,\star}^{r-2}(Z,\bar{Z})\right)-\partial_{\bar{Z}}\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})+\partial_{Z}\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z})=0 \quad (2.43)$$

Since $\partial_{\bar{Z}}$ and ∂_{Z} commute with $\widehat{\delta}$, then $\partial_{\bar{Z}}\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})$ and $\partial_{Z}\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z})$ are elements of the same space, so the only possibility to verify (2.43) is:

$$\widehat{\delta}\left(\Lambda_{Z,\bar{Z},\star}^{r-2}(Z,\bar{Z})-\partial_{\bar{Z}}\widehat{\bar{\mathcal{J}}}_{\bar{Z},0,\star}^{r-2}(Z,\bar{Z})+\partial_{\bar{Z}}\widehat{\mathcal{J}}_{Z,0,\star}^{r-2}(Z,\bar{Z})\right)=0 \quad (2.44)$$

$$-\partial_{\bar{Z}}\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})+\partial_{Z}\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z})=0 \quad (2.45)$$

Furthermore, since the $\widehat{\delta}$ cohomology does not depend on the external negative charged fields, solving (2.44) gives:

$$\left(\Lambda_{Z,\bar{Z},\star}^{r-2}(Z,\bar{Z})-\partial_{Z}\widehat{\bar{\mathcal{J}}}_{\bar{Z},0,\star}^{r-2}(Z,\bar{Z})+\partial_{\bar{Z}}\widehat{\mathcal{J}}_{Z,0,\star}^{r-2}(Z,\bar{Z})\right)=\widehat{\delta}\widehat{\Lambda}_{Z,\bar{Z},\star}^{r-3}(Z,\bar{Z}) \quad (2.46)$$

so the final decomposition writes:

$$\begin{aligned}
\mathcal{J}_{0,\star}^{\natural r}(z,\bar{z}) &= \gamma\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})+\bar{\gamma}\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z}) \\
&+ \delta\left(\gamma\bar{\gamma}\widehat{\Lambda}_{Z,\bar{Z},\star}^{r-3}(Z,\bar{Z})\right. \\
&+ \bar{\gamma}\widehat{\bar{\mathcal{J}}}_{\bar{Z},0,\star}^{r-2}(Z,\bar{Z})+\gamma\widehat{\mathcal{J}}_{Z,0,\star}^{r-2}(Z,\bar{Z}) \\
&+ \left.\widehat{\mathcal{J}}_{0,\star}^{r-1}(z,\bar{z})\right)
\end{aligned}$$
$$(2.47)$$

We emphasize that, in the "big" coordinates, the current $\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})$ will always transform as a scalar quantity, despite of its tensorial $\star$ content, that is:

$$\delta\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z})=(\gamma\partial_{Z}+\bar{\gamma}\partial_{\bar{Z}})\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) \quad (2.48)$$

On the other hand, introducing the local quantities:

$$\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z})\equiv\lambda\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) \quad (2.49)$$

$$\bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z})\equiv\bar{\lambda}\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z}) \quad (2.50)$$

they will transform in the "little" c ghosts as:

$$\delta \mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) = (c\partial + \bar{c}\bar{\partial})\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z})$$
$$+ (\partial c + \mu\partial\bar{c})\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) \tag{2.51}$$

so that it is a (1,0) tensor, while, going to the $C(z,\bar{z})$ ghosts, we get:

$$\delta \mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) = \partial\Big(C\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z})\Big)$$
$$+ \Big(\bar{\partial}\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) - \partial(\mu\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}))\Big)\frac{(\bar{C}(z,\bar{z}) - \bar{\mu}(z,\bar{z})C(z,\bar{z}))}{(1-\mu\bar{\mu})} \tag{2.52}$$

Similarly for its c.c. counterpart:

$$\delta \bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) = (c\partial + \bar{c}\bar{\partial})\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z})$$
$$+ (\bar{\partial}\bar{c} + \bar{\mu}\bar{\partial}c)\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) \tag{2.53}$$

$$\delta \bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) = \bar{\partial}\Big(\bar{C}\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z})\Big)$$
$$+ \Big(\partial\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) - \bar{\partial}(\bar{\mu}\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}))\Big)\frac{(C(z,\bar{z}) - \mu(z,\bar{z})\bar{C}(z,\bar{z}))}{(1-\mu\bar{\mu})} \tag{2.54}$$

(it is a (0,1) tensor)

So we can rewrite (2.47) as:

$$\mathcal{J}^{\natural r}_{0,\star}(z,\bar{z}) = C(z,\bar{z})\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) + \bar{C}(z,\bar{z})\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z})$$
$$+ \delta\Big(C(z,\bar{z})\bar{C}(z,\bar{z})\lambda\bar{\lambda}\widehat{\Lambda}^{r-3}_{Z,\bar{Z},\star}(Z,\bar{Z})$$
$$+ \bar{C}(z,\bar{z})\bar{\lambda}\widehat{\bar{\mathcal{J}}}^{r-2}_{\bar{Z},0,\star}(Z,\bar{Z})$$
$$+ C(z,\bar{z})\lambda\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\mathcal{J}}^{r-1}_{0,\star}(z,\bar{z})\Big) \tag{2.55}$$

A complete description in terms of the c's can be achieved introducing:

$$\mathcal{J}^{\natural r-1}_{0z,\star}(z,\bar{z}) = \mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) + \bar{\mu}\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) \tag{2.56}$$

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$$\mathcal{J}^{\natural\,r-1}_{0\bar{z},\star}(z,\bar{z}) = \bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) + \mu \mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) \tag{2.57}$$

and (2.55) then reads:

$$
\begin{aligned}
\mathcal{J}^{\natural\,r}_{0,\star}(z,\bar{z}) \;=\;& c(z,\bar{z})\mathcal{J}^{\natural\,r-1}_{0z,\star}(z,\bar{z}) + \bar{c}(z,\bar{z})\mathcal{J}^{\natural\,r-1}_{0\bar{z},\star}(z,\bar{z}) \\
+\;& \delta\Big(c(z,\bar{z})\bar{c}(z,\bar{z})(1-\mu\bar{\mu})\lambda\bar{\lambda}\widehat{\Lambda}^{r-3}_{Z,\bar{Z},\star}(Z,\bar{Z}) \\
+\;& (\bar{c}(z,\bar{z}) + \bar{\mu}(z,\bar{z})c(z,\bar{z}))\bar{\lambda}\widehat{\bar{\mathcal{J}}}^{\,r-2}_{\bar{Z},0,\star}(Z,\bar{Z}) \\
+\;& (c(z,\bar{z}) + \mu(z,\bar{z})\bar{c}(z,\bar{z}))\lambda(z,\bar{z})\widehat{\mathcal{J}}^{r-2}_{Z,0,\star}(Z,\bar{Z}) + \widehat{\mathcal{J}}^{\,r-1}_{0,\star}(z,\bar{z})\Big)
\end{aligned}
\tag{2.58}
$$

Therefore our final result for the currents (2.10), by specializing r=1, writes:

$$\mathcal{J}^{0}_{1,\star}(z,\bar{z}) = -\left(dz\frac{D}{Dc(z,\bar{z})} + d\bar{z}\frac{D}{D\bar{c}(z,\bar{z})} \right)\mathcal{J}^{\natural\,1}_{0,\star}(z,\bar{z}) + d\widehat{\mathcal{J}}^{0}_{0,\star}(z,\bar{z}) + \delta\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z}) \tag{2.59}$$

$$= \mathcal{S}^{\natural,0}_{z,0,\star}(z,\bar{z})(dz+\mu d\bar{z}) + \bar{\mathcal{S}}^{\natural,0}_{\bar{z},0,\star}(z,\bar{z})(d\bar{z}+\bar{\mu}dz) + d\widehat{\mathcal{J}}^{0}_{0,\star}(z,\bar{z}) + \delta\widehat{\mathcal{J}}^{-1}_{1,\star}(z,\bar{z}) \tag{2.60}$$

$$\equiv \mathcal{J}^{\natural\,0}_{0z,\star}(z,\bar{z})dz + \mathcal{J}^{\natural\,0}_{0\bar{z},\star}(z,\bar{z})d\bar{z} + d\widehat{\mathcal{J}^{0}_{0,\star}}(z,\bar{z}) + \delta\widehat{\mathcal{J}^{-1}_{1,\star}}(z,\bar{z}) \tag{2.61}$$

or, by covariance:

$$\mathcal{J}^{0}_{1,\star}(z,\bar{z}) = \mathcal{J}^{\natural,0}_{Z,0,\star}(Z,\bar{Z})dZ + \bar{\mathcal{J}}^{\natural,0}_{\bar{Z},0,\star}(Z,\bar{Z})d\bar{Z} + d\widehat{\mathcal{J}}^{0}_{Z,0,\star}(Z,\bar{Z}) + \delta\widehat{\mathcal{J}}^{-1}_{Z,0,\star}(Z,\bar{Z}) \tag{2.62}$$

Finally we want to remark that $\delta\mathcal{J}^{\natural\,r}_{0,\star}(z,\bar{z}) = 0$ implies, by using the decomposition (2.47) :

$$
\begin{aligned}
\gamma\bar{\gamma}\Big(-\partial_{\bar{Z}}\mathcal{J}^{\natural,r-1}_{Z,0,\star}(z,\bar{z}) &+ \partial_{Z}\bar{\mathcal{J}}^{\natural,r-1}_{\bar{Z},0,\star}(z,\bar{z}) \Big) \\
&= \frac{C(z,\bar{z})\bar{C}(z,\bar{z})}{(1-\mu(z,\bar{z})\bar{\mu}(z,\bar{z}))}\Big(\bar{\partial}\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z}) - \partial(\mu\mathcal{S}^{\natural,r-1}_{z,0,\star}(z,\bar{z})) \\
&\quad - \partial\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z}) + \bar{\partial}(\bar{\mu}\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\star}(z,\bar{z})) \Big) \\
&= c(z,\bar{z})\bar{c}(z,\bar{z})\Big(-\bar{\partial}\mathcal{J}^{\natural\,r-1}_{0z,\star}(z,\bar{z}) + \partial\mathcal{J}^{\natural\,r-1}_{0\bar{z},\star}(z,\bar{z}) \Big) \tag{2.63}
\end{aligned}
$$

the last equality tells us that that the diffeomorphism invariance will imply the current conservation both in the "big" index current $\mathcal{J}^{\natural,r-1}_{Z,0,\star}(z,\bar{z})$ and in the "little" index one $\mathcal{J}^{\natural\,r-1}_{0z,\star}(z,\bar{z})$, that is $d\mathcal{J}^{0}_{1,\star}(z,\bar{z}) = 0$ as before.

2.3 The charge definition and the holomorphic factorization of currents

The two dimensional character of the theory has an important consequence, since the current conservation condition can be inverted. In fact from the condition (2.45):

$$\left(-\partial_{\bar{Z}}\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) + \partial_Z\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z})\right) = 0 \tag{2.64}$$

it will formally follow:

$$\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) = (\partial_{\bar{Z}})^{-1}\partial_Z\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z}) \tag{2.65}$$

where the inverse operator $(\partial_{\bar{Z}})^{-1}$ can be defined, only in two dimension, using the Cauchy theorem:

$$\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) = \int_{\mathbf{C}} dW \wedge d\bar{W} \frac{\partial_W \bar{\mathcal{J}}_{\bar{W},0,\star}^{\natural,r-1}(W,\bar{W})}{W - Z}$$

$$= \int_{\mathbf{C}} dW \wedge d\bar{W} \partial_W\left(\frac{\bar{\mathcal{J}}_{\bar{W},0,\star}^{\natural,r-1}(W,\bar{W})}{W - Z}\right) - \int_{\mathbf{C}} dW \wedge d\bar{W} \partial_W\left(\frac{1}{W - Z}\right)\bar{\mathcal{J}}_{\bar{W},0,\star}^{\natural,r-1}(W,\bar{W})$$

$$= \partial_Z \int_{\mathbf{C}} dW \wedge d\bar{W} \frac{\bar{\mathcal{J}}_{\bar{W},0,\star}^{\natural,r-1}(W,\bar{W})}{W - Z} + \int_{\partial W} \frac{\bar{\mathcal{J}}_{\bar{W},0,\star}^{\natural,r-1}(W,\bar{W})}{W - Z} \tag{2.66}$$

and on a manifold without boundary it reduces to:

$$\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) = (\partial_Z)\Omega_{0,\star}^{\natural,r-1}(Z,\bar{Z}) \tag{2.67}$$

so no charge $Q_\star$ can be obtained unless:

$$\partial_{\bar{Z}}\mathcal{J}_{Z,0,\star}^{\natural,r-1}(Z,\bar{Z}) = 0$$
$$\partial_Z\bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,r-1}(Z,\bar{Z}) = 0 \tag{2.68}$$

hence, holomorphicity in Z (big!) is a necessary condition in order
to get charges.

In the z-frame, by (2.49) and (2.50), the existence of charges is assured only if we require the supplementary conditions:

$$\bar{\partial}\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}) = \partial(\mu\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}))$$
$$\partial\bar{\mathcal{S}}_{z,0,\star}^{\natural,r-1}(z,\bar{z}) = \bar{\partial}(\bar{\mu}\bar{\mathcal{S}}_{z,0,\star}^{\natural,r-1}(z,\bar{z})) \tag{2.69}$$

Accordingly, the δ variations (2.52) (2.54) read:

$$\delta \mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}) = \partial\Big(C(z,\bar{z})\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z})\Big)$$

$$\tag{2.70}$$

$$\delta \bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z}) = \bar{\partial}\Big(\bar{C}(z,\bar{z})\bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z})\Big) \tag{2.71}$$

we remark that if we define the currents $\mathcal{S}_{z,0}^{\natural,r-1}(z,\bar{z})$ and $\bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z})$ from their previous BRS variations, then the conditions (2.69) will derive from the required conditions (2.6):

$$
\begin{aligned}
\partial \bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z}) &= \Big\{\delta, \frac{D}{Dc(z,\bar{z})}\Big\}\bar{\mathcal{S}}_{\bar{z},0,\star}^{\natural,r-1}(z,\bar{z}) \\
&= \bar{\partial}(\bar{\mu}\bar{\mathcal{S}}_{z,0,\star}^{\natural,r-1}(z,\bar{z}))
\end{aligned}
\tag{2.72}
$$

$$
\begin{aligned}
\bar{\partial} \mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}) &= \Big\{\delta, \frac{D}{D\bar{c}(z,\bar{z})}\Big\}\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}) \\
&= \partial(\mu\mathcal{S}_{z,0,\star}^{\natural,r-1}(z,\bar{z}))
\end{aligned}
\tag{2.73}
$$

In the Z coordinates these solutions will imply:

$$
\begin{aligned}
\bar{\partial}(\lambda \mathcal{J}_{Z,0,\star}^{\natural,0}(Z,\bar{Z})) - \partial(\mu\lambda \mathcal{J}_{Z,0,\star}^{\natural,0}(Z,\bar{Z})) &= 0 \\
\partial(\bar{\lambda} \bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,0}(Z,\bar{Z})) - \bar{\partial}(\bar{\mu}\bar{\lambda} \bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,0}(Z,\bar{Z})) &= 0
\end{aligned}
\tag{2.74}
$$

that is:

$$
\begin{aligned}
\lambda\bar{\lambda}(1 - \mu\bar{\mu})\partial_{\bar{Z}} \mathcal{J}_{Z,0,\star}^{\natural,0}(Z,\bar{Z}) &= 0 \\
\lambda\bar{\lambda}(1 - \mu\bar{\mu})\partial_{Z} \bar{\mathcal{J}}_{\bar{Z},0,\star}^{\natural,0}(Z,\bar{Z}) &= 0
\end{aligned}
\tag{2.75}
$$

which means that $\mathcal{J}_{Z,0,\star}^{\natural,0}(Z,\bar{Z})$ is a holomorphic function iz Z.

This constraint can be imposed in a diff-invariant way by imposing:

$$
\begin{aligned}
\delta\Big(\gamma^{Z} \mathcal{J}_{Z,0,\star}^{0}(Z,\bar{Z})\Big) &= 0 \\
\delta\Big(\bar{\gamma}^{\bar{Z}} \bar{\mathcal{J}}_{\bar{Z},0,\star}^{0}(Z,\bar{Z})\Big) &= 0
\end{aligned}
\tag{2.76}
$$

or equivalently:

$$\delta\Big(C\mathcal{S}_{z,0,\star}^{0}(z,\bar{z})\Big) = 0 \tag{2.77}$$

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$$\delta\left(\bar{C}^{\bar{z}}\bar{\mathcal{S}}^0_{\bar{z},0,\star}(z,\bar{z})\right) = 0 \qquad (2.78)$$

Projecting (2.77) into underivated ghost factors we obtain:

$$c\bar{c}\left[-\bar{\partial}\mathcal{S}^0_{z,0,\star}(z,\bar{z}) + \partial\mu\mathcal{S}^0_{z,0,\star}(z,\bar{z}) + \mu\partial\mathcal{S}^0_{z,0,\star}(z,\bar{z})\right] = 0$$
$$c\left[\left(\partial c + \mu\partial\bar{c}\right)\mathcal{S}^0_{z,0,\star}(z,\bar{z}) - \hat{\delta}\mathcal{S}^0_{z,0,\star}(z,\bar{z})\right] = 0$$
$$\bar{c}\mu\left[\left(\partial c + \mu\partial\bar{c}\right)\mathcal{S}^0_{z,0,\star}(z,\bar{z}) - \hat{\delta}\mathcal{S}^0_{z,0,\star}(z,\bar{z})\right] = 0$$

$$(2.79)$$

and similarly for (2.78)

We recall that the $\star$ index will indicate arbitrary "big" Z $\bar{Z}$ indices, so the switching to "little" coordinates from $\mathcal{S}^0_{z,0,\star}(z,\bar{z}) \equiv \mathcal{S}^0_{z,0,Z^n,\bar{Z}^m}(z,\bar{z})$ is done with a suitable λ and $\bar{\lambda}$ rescaling.

$$\mathcal{S}^0_{z,0,Z^n,\bar{Z}^m}(z,\bar{z}) = \frac{\mathcal{S}^0_{z,0,z^n}(z,\bar{z})}{\lambda^n\bar{\lambda}^m}$$
$$\bar{\mathcal{S}}^0_{\bar{z},0,Z^n,\bar{Z}^m}(z,\bar{z}) = \frac{\bar{\mathcal{S}}^0_{\bar{z},0,\bar{z}^n}(z,\bar{z})}{\lambda^n\bar{\lambda}^m} \qquad (2.80)$$

It is easy to verify that the previous currents do not verify the local current conservation in μ and $\bar{\mu}$ (2.69) unless a particular tensorial content is realized: in particular this is achieved in (2.79) only if $\mathcal{S}^0_{z,0,\star}(z,\bar{z})$ will contain only Z indices and $\bar{\mathcal{S}}^0_{\bar{z},0,\star}(z,\bar{z})$ only $\bar{Z}$ ones, signature of the holomorphicity in Z.

So we obtain local currents $\mathcal{S}^0_{z,0,z^n}(z,\bar{z})$ $\bar{\mathcal{S}}^0_{\bar{z},0,\bar{z}^n}(z,\bar{z})$ defined as:

$$\mathcal{S}^0_{z,0,Z^n}(z,\bar{z}) = \frac{\mathcal{S}^0_{z,0,z^n}(z,\bar{z})}{\lambda^n}$$
$$\bar{\mathcal{S}}^0_{\bar{z},0,\bar{Z}^n}(z,\bar{z}) = \frac{\bar{\mathcal{S}}^0_{\bar{z},0,\bar{z}^n}(z,\bar{z})}{\bar{\lambda}^n} \qquad (2.81)$$

The constraint equations will be so modified:

$$c\bar{c}\left[-\bar{\partial}\mathcal{S}^0_{z,0,z^n}(z,\bar{z}) + (n+1)\partial\mu\mathcal{S}^0_{z,0,z^n}(z,\bar{z}) + \mu\partial\mathcal{S}^0_{z,0,z^n}(z,\bar{z})\right] = 0$$
$$c\left[(n+1)\left(\partial c + \mu\partial\bar{c}\right)\mathcal{S}^0_{z,0,z^n}(z,\bar{z}) - \hat{\delta}\mathcal{S}^0_{z,0,z^n}(z,\bar{z})\right] = 0$$
$$\bar{c}\mu\left[(n+1)\left(\partial c + \mu\partial\bar{c}\right)\mathcal{S}^0_{z,0,z^n}(z,\bar{z}) - \hat{\delta}\mathcal{S}^0_{z,0,z^n}(z,\bar{z})\right] = 0$$

$$(2.82)$$

and similarly for the c.c.

It is important to note that the current $\mathcal{S}^0_{z,0,z^n}(z,\bar{z})$ has a definite covariance property in the sense that is a "true" $(n+1),0$ tensor, it has a spin value $(n+1)$.

In terms of holomorphic ghosts we get:

$$\delta\mathcal{S}^{\natural,r-1}_{z,0,z^n}(z,\bar{z}) = \partial\Big(C(z,\bar{z})\mathcal{S}^{\natural,r-1}_{z,0,z^n}(z,\bar{z})\Big) + n\partial C(z,\bar{z})\mathcal{S}^{\natural,r-1}_{z,0,z^n} \tag{2.83}$$

$$\delta\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\bar{z}^n}(z,\bar{z}) = \bar{\partial}\Big(\bar{C}(z,\bar{z})\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\bar{z}^n}(z,\bar{z})\Big) + n\bar{\partial}\bar{C}(z,\bar{z})\bar{\mathcal{S}}^{\natural,r-1}_{\bar{z},0,\bar{z}^n}(z,\bar{z}) \tag{2.84}$$

3 The 1-forms in Lagrangian local Quantum Field Theory

3.1 The Classical Level

The previous Section has introduced, from a heuristic point of view the descent Equations which define the 1-forms. Moreover these equations play an important role in the dynamics; so they have to be embedded in a Lagrangian model whose quantum extension will provide informations concerning the renormalization of those currents.

We have to introduce an invariant Classical Action $\mathbf{\Gamma}^{Cl}_0$ such that:

$$\delta_0\mathbf{\Gamma}^{Cl}_0 = 0 \tag{3.1}$$

In [1] we have shown that the more general invariant Classical Action under diffeomorphisms takes the form:

$$
\begin{aligned}
\mathbf{\Gamma}^{Cl}_0 &= \int\Big(\sum_j a_j[\phi_{1-j,0}(\bar{\partial} - \mu\partial - j\partial\mu)\phi_{j,0} + c.c.](z,\bar{z})d\bar{z}\wedge dz \\
&+ (1-\mu\bar{\mu})(\phi_{1,0}\phi_{0,1} + c.c)(z,\bar{z})d\bar{z}\wedge dz \\
&+ b_j[\phi_{-j,-j}(\partial - \bar{\mu}\bar{\partial} - j\bar{\partial}\bar{\mu})\phi_{0,j}\frac{\bar{\partial} - \mu\partial - j\partial\mu}{1-\mu\bar{\mu}}\phi_{j,0}](z,\bar{z})d\bar{z}\wedge dz \\
&+ \sum_{n\geq 0}\Big[c_n[(\phi_{00})^n(\partial - \bar{\mu}\bar{\partial})\phi_{00}\frac{\bar{\partial} - \mu\partial}{1-\mu\bar{\mu}}\phi_{00}](z,\bar{z}) + c.c.\Big]d\bar{z}\wedge dz\Big)
\end{aligned}
\tag{3.2}
$$

We shall treat here, for the sake of simplicity, the spin zero case; in this case the most general invariant Lagrangian reads:

$$\mathbf{\Gamma}^{Cl}_0 = \int\Big(\sum_{n\geq 0}\Big[c_n[(\phi_{00})^n(\partial - \bar{\mu}\bar{\partial})\phi_{00}\frac{\bar{\partial} - \mu\partial}{1-\mu\bar{\mu}}\phi_{00}](z,\bar{z})d\bar{z}\wedge dz\Big]\Big)$$

We remark that, due to the dimensionless character of the scalar field, the only diff-invariance requirement will imply an infinite number of interaction terms at the Classical level, raising a lot

of problems on the physical meaning of the model; anyhow several criteria can be established on the resummation of the interacting part, which will involve particular addition conditions on the definition on the model which would not destroy the reparametrization invariance. These aspects do not compromise our treatment which will hold validity for all these classes of models.

According to the general prescription, we have to introduce the B.R.S. variations coupled to Φ, Π negative charged external fields.

Furthermore, as said before, we put into the dynamics the 1-forms current coupled to external fields.

$$\mathbf{\Gamma}^{Cl} = \mathbf{\Gamma}^{Cl}_0 + \mathbf{\Gamma}^{Cl}_1 + \mathbf{\Gamma}^{Cl}_2$$

(3.3)

where the "source" Action reads:

$$\begin{aligned}
\mathbf{\Gamma}^{Cl}_1 &= \int d\bar{z} \wedge dz \Big(\Big(J_{1,1}(z,\bar{z})\phi_{0,0}(z,\bar{z}) + \gamma_{1,1}(z,\bar{z})s\phi_{0,0}(z,\bar{z}) \Big) \\
&+ \eta(z,\bar{z})s\mu(z,\bar{z}) + \bar{\eta}(z,\bar{z})s\bar{\mu}(z,\bar{z}) + \zeta(z,\bar{z})sc(z,\bar{z}) + \bar{\zeta}(z,\bar{z})s\bar{c}(z,\bar{z}) \Big) \Big)
\end{aligned}$$

(3.4)

whith:

$$\begin{aligned}
s\mu &= (c\cdot\partial)\mu - \mu(\partial c + \mu\partial\bar{c}) + \bar{\partial}c + \mu\bar{\partial}\bar{c} \\
sc &= (c\cdot\partial)c \\
s\phi_{0,0} &= (c\cdot\partial)\phi_{0,0} ,
\end{aligned}$$

(3.5)

and of course the complex conjugate expressions and the "current" Action :

$$\begin{aligned}
\mathbf{\Gamma}^{Cl}_2 &= \int d\bar{z} \wedge dz \Big(\rho^{z^n}(z,\bar{z})\Lambda^{-1}_{z,\bar{z},z^n}(z,\bar{z}) + \bar{\rho}^{\bar{z}^n}(z,\bar{z})\bar{\Lambda}^{-1}_{z,\bar{z},\bar{z}^n}(z,\bar{z}) \\
&+ \beta^{z^n}_{\bar{z}}(z,\bar{z})\mathcal{S}^0_{z,0,z^n}(z,\bar{z}) + \bar{\beta}^{\bar{z}^n}_z(z,\bar{z})\bar{\mathcal{S}}^0_{\bar{z},0,\bar{z}^n}(z,\bar{z}) \Big)
\end{aligned}$$

(3.6)

We have to impose the invariant condition:

$$\delta_0\mathbf{\Gamma}^{Cl} = 0$$

(3.7)

The U.V. dimensions of the constituents of the model are:

$$[\partial] = [\bar{\partial}] = 1$$

(3.8)

$$[\phi_{0,0}] = 0$$

(3.9)

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$$[\mu] = [c] = [\bar{c}] = 0$$

$$(3.10)$$

$$[\gamma_{1,1}] = 1$$

$$(3.11)$$

$$[\zeta] = [\eta] = [\bar{\eta}] = 1$$

$$(3.12)$$

$$[\rho^{z^n}] = [\beta_{\bar{z}}^{z^n}] = [\bar{\beta}_z^{\bar{z}^n}] = 1 - n$$

$$(3.13)$$

$$[\mathcal{S}^0_{z,0,z^n}(z,\bar{z})] = 1 + n$$

$$(3.14)$$

The external fields coupled to the holomorphic currents are introduced in the Lagrangian, by fixing their variations in order to get the descent equations seen in the previous Section: So we have:

$$s\rho^{z^n}(z,\bar{z}) \;=\; (c{\cdot}\partial)\rho^{z^n}(z,\bar{z}) - n(\partial c(z,\bar{z}) + \mu\partial\bar{c}(z,\bar{z}))\rho^{z^n}(z,\bar{z})$$

$$(3.15)$$

$$
\begin{aligned}
s\beta_z^{z^n}(z,\bar{z}) \;=\;& (c{\cdot}\partial)\beta_z^{z^n}(z,\bar{z}) \\
&+(\partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z}))\beta_z^{z^n}(z,\bar{z}) \\
&-(n+1)(\partial c(z,\bar{z}) + \mu\partial\bar{c})\beta_z^{z^n}(z,\bar{z}) \\
&+\bar{\partial}\rho^{z^n}(z,\bar{z}) - \mu\partial\rho^{z^n}(z,\bar{z}) + n\partial\mu\rho^{z^n}(z,\bar{z})
\end{aligned}
$$

$$(3.16)$$

and their c.c. expressions. That is, if we define:

$$\rho^{Z^n}(Z,\bar{Z}) = (\lambda)^n \rho^{z^n}(z,\bar{z})$$

$$(3.17)$$

$$\beta_Z^{Z^n}(Z,\bar{Z}) = \frac{\lambda^{(n+1)}}{\lambda\bar{\lambda}(1-\mu\bar{\mu})}\beta_z^{z^n}(z,\bar{z})$$

$$(3.18)$$

we get the following descent equations for the sources:

$$\hat{\delta}\beta_Z^{Z^n}(Z,\bar{Z}) = \partial_{\bar{Z}}\rho^{Z^n}(Z,\bar{Z})$$
$$\hat{\delta}\rho^{Z^n}(Z,\bar{Z}) = 0$$

$$(3.19)$$

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the introduction of the previous fields allows us to reproduce at the Lagrangian level the right properties of the holomorphic currents $\mathcal{S}^0_{\bar{z},0,z^n}(z,\bar{z})$ at the classical level, when the simmetry is preserved.

The role of the ρ^{z^n} field, as inhomogeneous part of the β tranformations, is of prime importance: it will fix the current conservation (2.69). The BRS philosophy forces us to fix their covariance properties, so the $\Lambda_{z,\bar{z},z^n}(z,\bar{z})$ term is a priori needed at the tree level: we shall show that this term is unessential at the Classical level, but, on the other hand, is fundamental at the quantum level.

The BRS operator is defined:

$$
\begin{aligned}
\delta_0 &= \int d\bar{z} \wedge dz \left(\frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \gamma_{1,1}(z,\bar{z})} \frac{\delta}{\delta \phi_{0,0}(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \phi_{0,0}(z,\bar{z})} \frac{\delta}{\delta \gamma_{1,1}(z,\bar{z})} \right. \\
&+ \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \eta(z,\bar{z})} \frac{\delta}{\delta \mu(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \mu(z,\bar{z})} \frac{\delta}{\delta \eta(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \bar{\eta}(z,\bar{z})} \frac{\delta}{\delta \bar{\mu}(z,\bar{z})} \\
&+ \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \bar{\mu}(z,\bar{z})} \frac{\delta}{\delta \bar{\eta}(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \zeta(z,\bar{z})} \frac{\delta}{\delta c(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta c(z,\bar{z})} \frac{\delta}{\delta \zeta(z,\bar{z})} \\
&+ \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \bar{\zeta}(z,\bar{z})} \frac{\delta}{\delta \bar{c}(z,\bar{z})} + \frac{\delta \mathbf{\Gamma}^{Cl}}{\delta \bar{c}(z,\bar{z})} \frac{\delta}{\delta \bar{\zeta}(z,\bar{z})} \\
&+ s\rho^{z^n}(z,\bar{z}) \frac{\delta}{\delta \rho^{z^n}(z,\bar{z})} + s\bar{\rho}^{\bar{z}^n}(z,\bar{z}) \frac{\delta}{\delta \bar{\rho}^{\bar{z}^n}(z,\bar{z})} + s\beta_z^{z^n}(z,\bar{z}) \frac{\delta}{\delta \beta_z^{z^n}(z,\bar{z})} \\
&+ \left. s\bar{\beta}_{\bar{z}}^{\bar{z}^n}(z,\bar{z})) \frac{\delta}{\delta \bar{\beta}_{\bar{z}}^{\bar{z}^n}(z,\bar{z})} \right)
\end{aligned}
\tag{3.20}
$$

with

$$
s\gamma_{1,1}(z,\bar{z}) = \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \phi_{0,0}(z,\bar{z})} + (c \cdot \partial)\gamma_{1,1}(z,\bar{z}) + (\partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z}))\gamma_{1,1}(z,\bar{z})
\tag{3.21}
$$

and

$$
s\eta(z,\bar{z}) = \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \mu(z,\bar{z})} + (c \cdot \partial)\eta(z,\bar{z}) + (\partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z}))\eta(z,\bar{z})
$$
$$
+(\partial c(z,\bar{z}) + 2\mu(z,\bar{z})\partial\bar{c}(z,\bar{z}) - \bar{\partial}\bar{c}(z,\bar{z}))\eta(z,\bar{z})
\tag{3.22}
$$

$$
s\zeta(z,\bar{z}) = -\gamma_{1,1}(z,\bar{z})\partial\phi_{0,0}(z,\bar{z}) - \eta(z,\bar{z})\partial\mu(z,\bar{z}) - \partial(\mu(z,\bar{z})\eta(z,\bar{z})) - \bar{\partial}\eta(z,\bar{z})
$$
$$
-\bar{\eta}(z,\bar{z})\partial\bar{\mu}(z,\bar{z}) - \partial(\mu^2(z,\bar{z})\bar{\eta}(z,\bar{z})) + \zeta(z,\bar{z})\partial c(z,\bar{z}) - \partial(\zeta(z,\bar{z})c(z,\bar{z})) - \bar{\partial}(\zeta\bar{c}(z,\bar{z}))
\tag{3.23}
$$

So we can write the current Ward identities, coming from the B.R.S.

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variation of the external sources $\rho^{z^n}(z,\bar{z}), \beta_{\bar{z}}^{z^n}(z,\bar{z})$. They reproduce in a functional approach the descent equations just encountered in the previous Section, written in terms of $\Lambda_{z,\bar{z},z^n}(z,\bar{z}), \mathcal{S}_{z,0,z^n}^{r-1}(z,\bar{z})$ and their c.c.

$$
\frac{\delta}{\delta\rho^{z^n}(z,\bar{z})}\Big(\delta_0\boldsymbol{\Gamma}^{Cl}\Big) = \delta\frac{\delta_0\boldsymbol{\Gamma}^{Cl}}{\delta\rho^{z^n}(z,\bar{z})} - \partial\Big(c\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\rho^{z^n}(z,\bar{z})}\Big) - \bar{\partial}\Big(\bar{c}\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\rho^{z^n}(z,\bar{z})}\Big)
$$
$$
-n(\partial c(z,\bar{z}) + \mu\partial\bar{c}(z,\bar{z}))\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\rho^{z^n}(z,\bar{z})}
$$
$$
-\bar{\partial}\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}(z,\bar{z})} + \mu\partial\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}(z,\bar{z})} + (n+1)\partial\mu\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}(z,\bar{z})}
$$
$$
\equiv \Big(\delta_0 - c\partial - \bar{c}\bar{\partial} - \bar{\partial}\bar{c} - \partial c\Big)\Lambda_{z,\bar{z},z^n}(z,\bar{z})
$$
$$
-\bar{\partial}\mathcal{S}_{z,0,z^n}^0(z,\bar{z}) + (n+1)\partial\mu\mathcal{S}_{z,0,z^n}^0(z,\bar{z}) + \mu\partial\mathcal{S}_{z,0,z^n}^0(z,\bar{z}) = 0 \qquad (3.24)
$$

$$
\frac{\delta}{\delta\beta_{\bar{z}}^{z^n}(z,\bar{z})}\Big(\delta_0\boldsymbol{\Gamma}^{Cl}\Big) = \delta_0\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}^{z^n}(z,\bar{z})} - \partial\Big(c(z,\bar{z})\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}^{z^n}(z,\bar{z})}\Big) - \bar{\partial}\Big(\bar{c}(z,\bar{z})\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}^{z^n}(z,\bar{z})}\Big)
$$
$$
+(\partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z}))\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}^{z^n(z,\bar{z})}} - (n+1)(\partial c + \mu\partial\bar{c})\frac{\delta\boldsymbol{\Gamma}^{Cl}}{\delta\beta_{\bar{z}}^{z^n(z,\bar{z})}}
$$
$$
\equiv \Big(\hat{\delta} - c\partial - \bar{c}\bar{\partial} - (n+1)(\partial c + \mu\partial\bar{c})\Big)\mathcal{S}_{z,0,z^n}^{r-1}(z,\bar{z}) = 0
$$
$$
(3.25)
$$

Their solutions are carried out as before; introducing:

$$
\mathcal{S}_{Z,0,Z^n}^0(Z,\bar{Z}) = \frac{\mathcal{S}_{z,0,z^n}^0(z,\bar{z})}{\lambda^{(n+1)}}
$$
$$
\Lambda_{Z,\bar{Z},Z^n}(Z,\bar{Z}) = \frac{\Lambda_{z,\bar{z},z^n}(z,\bar{z})}{\lambda^{(n+1)}\bar{\lambda}(1-\mu\bar{\mu})}
$$
$$
(3.26)
$$

they become:

$$
\hat{\delta}\mathcal{S}_{Z,0,Z^n}^0(Z,\bar{Z}) = 0
$$
$$
\hat{\delta}\Lambda_{Z,\bar{Z},Z^n}(Z,\bar{Z}) = \partial_{\bar{Z}}\mathcal{S}_{Z,0,Z^n}^0(Z,\bar{Z})
$$
$$
(3.27)
$$

so:

$$
\mathcal{S}_{Z,0,Z^n}^0(Z,\bar{Z}) = \mathcal{S}_{Z,0,Z^n}^{0,\natural}(Z,\bar{Z}) + \hat{\delta}\widehat{\mathcal{S}}_{Z,0,Z^n}^{r-2}(Z,\bar{Z}) \qquad (3.28)
$$

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so (3.24) is rewritten as

$$\hat{\delta}\Big(\Lambda_{Z,\bar{Z},Z^n}(Z,\bar{Z}) - \partial_Z \hat{\mathcal{S}}^{r-2}_{Z,0,Z^n}(Z,\bar{Z})\Big) = \partial_{\bar{Z}}\mathcal{S}^{0,\natural}_{Z,0,Z^n}(Z,\bar{Z}) \tag{3.29}$$

But, since, $\partial_{\bar{Z}}\mathcal{S}^{0,\natural}_{Z,0,Z^n}(Z,\bar{Z})$ is an element of the $\hat{\delta}$-cohomology, the previous equation is consistent only if each term is identically zero: we have shown so that the diff invariance will imply the holomorphicity of $\mathcal{S}^{0,\natural}_{Z,0,Z^n}(Z,\bar{Z})$, that is:

$$\frac{\delta}{\delta\rho^{z^n}(z,\bar{z})}\Big(\delta_0\mathbf{\Gamma}^{Cl}\Big) = 0 \longrightarrow \partial_{\bar{Z}}\mathcal{S}^{0,\natural}_{Z,0,Z^n}(Z,\bar{Z}) = 0 \tag{3.30}$$

So the current conservation will derive from the inhomogeneous part of the β variation, so a priori we have to require:

$$(\bar{\partial}\rho^{z^n}(z,\bar{z}) - \mu\partial\rho^{z^n}(z,\bar{z}) + n\partial\mu\rho^{z^n}(z,\bar{z})) \neq 0. \tag{3.31}$$

the eq.(3.31) will imply:

$$\partial_{\bar{Z}}\rho^{Z^n}(z,\bar{z}) \neq 0 \tag{3.32}$$

On the other hand it is easy to realize from (3.24) (3.25) that in the quantum extension of the model, the possible ρ and β dependent anomalies will spoil the current conservation and their covariance properties respectively.

The B.R.S. approach consists in the study of the cohomology of the δ operator in the space of local functionals, the charge zero space will identify the Classical Action while the charge one will give the quantum anomalies.

This analysis has to be done as the one carried out in [1], where, in a similar way we have related the diff-mod $\mathbf{d}$ cohomolology space to the one of δ within the class of local functions.

Calling $\Delta^p_2(z,\bar{z})$ the more general element of the diff-mod $\mathbf{d}$ cohomolology and labeling with the $\natural$ index the δ cohomology elements we can find the 2-form extension of (2.7) as calculated in [1]

$$\Delta^p_2(z,\bar{z}) = \Delta^{\natural p}_2(z,\bar{z}) - \frac{D}{Dc(z,\bar{z})}\frac{D}{D\bar{c}(z,\bar{z})}\Delta^{\natural p+2}_0(z,\bar{z})dz \wedge d\bar{z}$$

$$\tag{3.33}$$
$$- \frac{D}{Dc(z,\bar{z})}\Delta^{\natural p+1}_1(z,\bar{z})dz - \frac{D}{D\bar{c}(z,\bar{z})}\Delta^{\natural p+1}_1(z,\bar{z})d\bar{z} - d\hat{\Delta}^p_{1,\star}(z,\bar{z}) + \delta\hat{\Delta}^{p-1}_{2,\star}(z,\bar{z}) \ ,$$

The novelty of this paper with respect to [1] consists in

introducing the currents inside the dynamics of the Lagrangian, and (as it is easy to realize from (3.24) (3.25)) the quantum extension of the model might generate ρ and β dependent anomalies which could spoil the current conservation and their covariance properties respectively, as already stated.

The next Section will investigate this possibility.

3.2   The Quantum Level

The quantum extension of the model has to be done as in our paper [1]; first of all we have to parametrize the anomaly as:

$$
\begin{aligned}
\delta\mathbf{\Gamma} &= \Delta \\
&= \int d\bar{z} \wedge dz \Delta^0(z,\bar{z}) + \sum_n \Big(\rho^{z^n}(z,\bar{z}) \Delta^{0,n}_{z,\bar{z},z^n}(z,\bar{z}) \\
&+ \bar{\rho}^{\bar{z}^n}(z,\bar{z}) \bar{\Delta}^{0,n}_{z,\bar{z},\bar{z}^n}(z,\bar{z}) + \bar{\beta}^{\bar{z}^n}_z(z,\bar{z}) \bar{\Delta}^{1,n}_{\bar{z}\bar{z}^n}(z,\bar{z}) \\
&+ \beta^{z^n}_{\bar{z}}(z,\bar{z}) \Delta^1_{z,z^n}(z,\bar{z}) \Big)
\end{aligned}
\tag{3.34}
$$

and $\Delta^{0,n}_{z,\bar{z},z^n}(z,\bar{z})$ has Φ, Π charge equal to zero, and $\Delta^{1,n}_{z,z^n}$ with charge one, they both have U.V. dimensions 2+n.

The hunting of anomalies has to be done as in [1], and within this model we show in the Appendix that:

Theorem *The ghost sectors (Φ-Π charge sectors) of the $\widetilde{\delta}$-cohomology in the space of analytic functions of the fields, where the fields λ and $\bar{\lambda}$ satisfy the equation (1.5), and completed with the terms $\{\ln\lambda, \ln\bar{\lambda}\}$, seen as independent fields depend only on terms containing underivated source ρ which multiply zero ghost sector elements of the cohomology.*

The zero ghost sector is, on the other hand, non-trivial only in the part which contains matter fields. Its elements will contain no free z and $\bar{z}$ indices, i.e. they are "scalar-like" quantities with respect to "little indices" but can hold the tensorial content with respect the "big indices" Z and $\bar{Z}$. A generic element of this space will be a $(h, \bar{h})$-conformal quantity of the form

$$
[f(\partial^m_Z \partial^n_{\bar{Z}} \phi_{0,0}(Z, \bar{Z}))]_{h,\bar{h}},
$$

where f is an analytic function, (polynomial).

So we are left with a ρ dependent anomaly:

$$
\Delta^{\natural}_{z\bar{z}}(z,\bar{z}) = \lambda\bar{\lambda}(1 - \mu\bar{\mu})\rho\Delta^{2\natural}_{Z\bar{Z}}(z,\bar{z})
\tag{3.35}
$$

but from the transformation laws (3.15) we argue that:

$$
\rho(z,\bar{z}) \approx \partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z})
\tag{3.36}
$$

so eq (3.35) recover an ordinary trace anomaly which can be reabsorbed by a counterterm

$$
\int d\bar{z} \wedge dz \rho(z,\bar{z})\phi_{0,0}(z,\bar{z})\gamma_{1,1}(z,\bar{z})
$$

Indeed the B.R.S. variation of the previous term gives the anomaly:

$$s \int d\bar{z} \wedge dz \rho(z,\bar{z}) \phi_{0,0}(z,\bar{z}) \gamma_{1,1}(z,\bar{z}) =$$

$$- \int d\bar{z} \wedge dz \rho(z,\bar{z}) \phi_{0,0}(z,\bar{z}) \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \phi_{0,0}(z,\bar{z})} \tag{3.37}$$

At this stage we have to reconsider the model and all the cohomolology calculations serve to recover every possible origins of anomalies: in Ref [1] the locality requirement, which forces the elimination of our vectorial space $\mathcal{V}$ of λ, $\bar{\lambda}$ (but more important of $\ln \lambda$ and $\ln \bar{\lambda}$) dependence, was the origin of the holomorphic anomaly, which is represented by $\Delta^0(z,\bar{z})$. In this paper we are involved in the $\beta_z^{z^n}(z,\bar{z})$ and $\rho^{z^n}(z,\bar{z})$ andtheir c.c. external dependent anomalies, so we have to analyze again their disappearance from the cohomology sectors.

We recall that the cancellation of the $\rho^{z^n}(z,\bar{z})$ anomalies (see Appendix) are governed by the term

$$(\bar{\partial} \rho^{z^n}(z,\bar{z}) - \mu \partial \rho^{z^n}(z,\bar{z}) + n \partial \mu \rho^{z^n}(z,\bar{z})) \neq 0. \tag{3.38}$$

so possible $\rho^{z^n}(z,\bar{z})$ dependent anomalies can appear in the slice where

$$(\bar{\partial} \rho^{z^n}(z,\bar{z}) - \mu \partial \rho^{z^n}(z,\bar{z}) + n \partial \mu \rho^{z^n}(z,\bar{z})) = 0. \tag{3.39}$$

that is:

$$\partial_{\bar{Z}} \rho^{Z^n}(Z,\bar{Z}) = 0 \tag{3.40}$$

If (3.39) is valid, the field $\beta_z^{z^n}(z,\bar{z})$ is a "true" tensor density

$$\begin{aligned}
s\beta_z^{z^n}(z,\bar{z}) &= (c \cdot \partial)\beta_z^{z^n}(z,\bar{z}) \\
&+ (\partial c(z,\bar{z}) + \bar{\partial}\bar{c}(z,\bar{z}))\beta_z^{z^n}(z,\bar{z}) \\
&- (n+1)(\partial c(z,\bar{z}) + \mu \partial \bar{c})\beta_z^{z^n}(z,\bar{z})
\end{aligned}$$

$$\tag{3.41}$$

Furthermore in this region the covariance properties of $\mathcal{S}_{Z,0,Z^n}^{0,\natural}(z,\bar{z})$ are not modified, due (3.25) (or if you prefer the second and third (2.82) condition), and no $\beta_z^{z^n}(z,\bar{z})$ dependent anomalies can appear.

With this constraint the BRS variation of $\rho^{z^n}(z,\bar{z})$ becomes

$$s\rho^{z^n}(z,\bar{z}) = C(z,\bar{z})\partial \rho^{z^n}(z,\bar{z}) - n\partial C(z,\bar{z})\rho^{z^n}(z,\bar{z}) \tag{3.42}$$

It is so evident that this is a sign of the holomorphic factorization, so the anomalies have to be searched as elements of the local cohomology of the δ operator as polynomial in $\rho^{z^n}(z,\bar{z})$ and its

∂ derivatives, with the remaining field content in order to get $\Phi.\Pi$ charge and U.V.dimension equal to 3.

$$\Delta^\natural(z,\bar{z}) = \partial_r \rho^{z^n}(z,\bar{z})\Delta_n^r(z,\bar{z})$$
$$(\partial_r \equiv \partial_1 \partial_2 \cdots \partial_r)$$

$$(3.43)$$

$$dim\Delta_n^r(z,\bar{z}) = 2 + n - r \tag{3.44}$$

so for $r \leq 2 + n$ the cocycle condition:

$$s\Delta^\natural(z,\bar{z}) = 0 \tag{3.45}$$

implies:

$$\partial_r\Big(C(z,\bar{z})\partial\rho^{z^n}(z,\bar{z}) - n\partial C(z,\bar{z})\rho^{z^n}(z,\bar{z})\Big) - \partial_r\rho^{z^n}(z,\bar{z})s\Delta_n^r(z,\bar{z}) = 0 \tag{3.46}$$

and then:

$$\sum_{j\leq r}\partial_s\rho^{z^n}(z,\bar{z})\left[\binom{r}{j}\Big(-\partial_j C(z,\bar{z})\delta_{r-j+1}^s + n\partial_{j+1}C(z,\bar{z})\delta_{r-j}^s\Big)\Delta_n^r(z,\bar{z}) - \delta_r^s s\Delta_n^r(z,\bar{z})\right] = 0 \tag{3.47}$$

In other words:

$$\binom{r}{j}\Big(-\partial_j C(z,\bar{z})\delta_{r-j+1}^s + n\partial_{j+1}C(z,\bar{z})\delta_{r-j}^s\Big)\Delta_n^r(z,\bar{z}) = \delta_r^s s\Delta_n^r(z,\bar{z}) \tag{3.48}$$

for each $s, n \geq 0$

For s=r

$$s\Delta_n^r(z,\bar{z}) = (n-r)\partial C(z,\bar{z})\Delta_n^r(z,\bar{z}) \tag{3.49}$$

and for $s \neq r$

$$\sum_{j\leq r,j\neq 1}\binom{r}{j}\Big(-\partial_j C(z,\bar{z})\delta_{r-j+1}^s + n\partial_{j+1}C(z,\bar{z})\delta_{r-j}^s\Big)\Delta_n^r(z,\bar{z}) = 0 \tag{3.50}$$

Now the BRS operator will always contain $c\partial + \bar{c}\bar{\partial}\cdots$ which do not appear in (3.49) so the only solution will be for

$$n = r \tag{3.51}$$

and the anomaly takes the form

$$\Delta^\natural(z,\bar{z}) = \partial_n\rho^{z^n}(z,\bar{z})\Delta^n(z,\bar{z}) \tag{3.52}$$

where:

$$dim\Delta^n(z, \bar{z}) = 2 \tag{3.53}$$

for each n, and:

$$s\Delta^n(z, \bar{z}) = 0 \tag{3.54}$$

Furthermore from (3.50), for each n, we have: (3.50)

$$- C(z, \bar{z}) \begin{pmatrix} n-1 \\ 0 \end{pmatrix} \Delta^{n-1}(z, \bar{z})$$

$$+ \partial\partial C(z, \bar{z}) \left(- \begin{pmatrix} n+1 \\ 2 \end{pmatrix} + (n+1) \begin{pmatrix} n+1 \\ 1 \end{pmatrix} \right) \Delta^{n+1}(z, \bar{z})$$

$$+ \partial\partial\partial C(z, \bar{z}) \left(- \begin{pmatrix} n+2 \\ 3 \end{pmatrix} + (n+2) \begin{pmatrix} n+2 \\ 2 \end{pmatrix} \right) \Delta^{n+2}(z, \bar{z})$$

$$+ \cdots = 0 \tag{3.55}$$

where dots will contain $\Delta^n(z, \bar{z})$ ($n \geq 3$) with fixed dimensions equal to 2 which multiply terms $\partial^l C(z, \bar{z})$,($l \geq 4$) .

So the only cancellation mechanism relies on $\Phi\Pi$ tricks , and

by considering that $\Delta^n(z, \bar{z})$ does not contain any external field, power counting arguments forbid any solution; so:

$$\Delta^n(z, \bar{z}) = 0 \quad for \quad n \geq 2 \tag{3.56}$$

Elementary considerations show that the previous conditions are all verified only for

$$\Delta^n(z, \bar{z}) = C(z, \bar{z})\partial\partial C(z, \bar{z}), \quad n = 0, 1 \tag{3.57}$$

so we have

$$\Delta_0^\natural(z, \bar{z}) = \rho(z, \bar{z})C(z, \bar{z})\partial\partial C(z, \bar{z}) = \rho(z, \bar{z})s\Big(\partial C(z, \bar{z})\Big)$$

$$\Delta_1^\natural(z, \bar{z}) = \partial\rho^z(z, \bar{z})C(z, \bar{z})\partial\partial C(z, \bar{z}) = \partial\rho^z(z, \bar{z})s\Big(\partial C(z, \bar{z})\Big)$$

$$\tag{3.58}$$

since, in the case of (3.40), $\rho \approx \partial C(z, \bar{z})$ it is easy to realize that $\Delta_0^\natural(z, \bar{z})$ is a mimic of the Feigin Fuks cocycle [16].

In this framework we can verify that:

$$\Delta_0^{\natural}(z, \bar{z}) = s\Big(\rho(z, \bar{z})C(z, \bar{z})\partial \ln \lambda(z, \bar{z})\Big)$$

$$\Delta_1^{\natural}(z, \bar{z}) = s\Big(\partial \rho^z(z, \bar{z})C(z, \bar{z})\partial \ln \lambda(z, \bar{z}) - C(z, \bar{z})\partial \ln \lambda(z, \bar{z})\rho^z(z, \bar{z})\partial \ln \lambda(z, \bar{z})\Big)$$

$$(3.59)$$

so they are coboundaries in "non local " basis, while in a local ones the compensation mechanism is not possible and give rise to anomalies.

Finally calculations are concluded by deriving the Ward anomalies:

the Ward identity obstruction to the (1,0) current conservation takes the form:

$$\frac{\delta}{\delta\rho(z, \bar{z})} \int \frac{\partial}{\partial c(z', \bar{z}')} \frac{\partial}{\partial \bar{c}(z', \bar{z}')} \Delta_0^{\natural}(z', \bar{z}') dz' \wedge d\bar{z}' \approx \partial^2 \mu(z, \bar{z}) \tag{3.60}$$

and the (2,0) obstruction which corresponds to that of the energy momentum tensor reads:

$$\frac{\delta}{\delta\rho^z(z, \bar{z})} \int \frac{\partial}{\partial c(z', \bar{z},)} \frac{\partial}{\partial \bar{c}(z', \bar{z}')} \Delta_1^{\natural}(z', \bar{z}') dz' \wedge d\bar{z}' \approx \partial^3 \mu(z, \bar{z}) \tag{3.61}$$

4 Conclusions

In the present work we have studied the role of the diffeomorphism current both at the Classical and at the Quantum level by computing some specific cohomologies.

It has been shown, within the Beltrami parametrization of compex structures, that the holomorphic properties play a fundamental role in the dynamics of simple conformal models. This fact again infers the relevance of the complex structure of the Riemann surface on which the field theoretical model is built on.

The locality requirements govern deeply the occurrence of anomalies at the quantum level.

The study of diffeomorphism current is not completed here and deserves some more careful results in particular in the meaning of the anomalous Ward identities for correlation functions with diffeomorphism current insertions.

Appendix A The $\widetilde{\delta}$ cohomology

The previous results heavy rely on the calculation of the $\widetilde{\delta}$ operator on the space $\mathcal{V}$ of the local functions with positive power on the matter field $\phi_{0,0}$ and the $\Phi.\Pi.$ charged fields, and analitical in the Beltrami fields. These constaints are required on the basis of $\Phi.\Pi.$ charge superselection rules and Lagrangian contruction and play a relevant role, since the cohomology definition depends not only on the operator but on its domain too.For the construction given in the text the space $\mathcal{V}$ do not contain underivated $c(z, \bar{z})$, $\bar{c}(z, \bar{z})$ $\Phi.\Pi.$ ghosts.

The operator $\widetilde{\delta}$ is defined from δ as:

$$\widetilde{\delta} = \delta - (c(z, \bar{z}) \cdot \partial) = \delta - c(z, \bar{z}) \Big\{ \delta, \frac{D}{Dc(z, \bar{z})} \Big\} - \bar{c}(z, \bar{z}) \Big\{ \delta, \frac{D}{D\bar{c}(z, \bar{z})} \Big\} , \qquad (A.1)$$

where:

$$
\begin{aligned}
\delta = {} & sZ(z, \bar{z}) \frac{D}{DZ(z, \bar{z})} + s\bar{Z}(z, \bar{z}) \frac{D}{D\bar{Z}(z, \bar{z})} \\[2mm]
& + \sum_{m,n \geq 0} \Bigg(\partial^m \bar{\partial}^n s\phi_{j,\bar{j}}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \phi_{j,\bar{j}}(z, \bar{z})} + \partial^m \bar{\partial}^n s\mu(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \mu(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\bar{\mu}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\mu}(z, \bar{z})} + \partial^m \bar{\partial}^n s\lambda(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \lambda(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\bar{\lambda}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\lambda}(z, \bar{z})} + \partial^m \bar{\partial}^n s\gamma(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \gamma(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\eta(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \eta(z, \bar{z})} + \partial^m \bar{\partial}^n s\bar{\eta}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\eta}(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\zeta(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \zeta(z, \bar{z})} + \partial^m \bar{\partial}^n s\bar{\zeta}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\zeta}(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\rho(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \rho(z, \bar{z})} + \partial^m \bar{\partial}^n s\beta(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \beta(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n s\bar{\beta}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\beta}(z, \bar{z})} + \partial^m \bar{\partial}^n s\bar{\rho}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\rho}(z, \bar{z})} \\[2mm]
& + \partial^m \bar{\partial}^n sc(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n c(z, \bar{z})} + \partial^m \bar{\partial}^n s\bar{c}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{c}(z, \bar{z})} \Bigg) .
\end{aligned}
\qquad (A.2)
$$

We recall that $\widetilde{\delta}$ acts on the space on the fields and their derivatives considered as independent coordinates, as they stand for local Fock representation of the model.

The Spectral Sequencie analysys [13, 14, 15] is a "perturbative-like" method method which allows to recover, by recursion, a space which is isomorphic to the cohomology one.

First of all an adjoint procedure is introduced into the game (just copying the Fock-like creation and destruction procedure [14]) by the formal replacement [15] of the formal derivative with respect to the field and their derivatives by the formal multiplication with respect the same quantities and vice versa.

Introducing the self-adjoint operator:

$$\nu = \sum_{m,n \geq 0, \; m+n \geq 1} (m+n) \left(\partial^m \bar{\partial}^n c(z,\bar{z}) \frac{D}{D\partial^m \bar{\partial}^n c(z,\bar{z})} + \partial^m \bar{\partial}^n \bar{c}(z,\bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{c}(z,\bar{z})} \right) \; , \quad \text{(A.3)}$$

whose eigenvalues provide the counting of the order of the ghost derivatives; so the space $\mathcal{V}$ can be decomposed into a direct sum of subspaces; futhermore $\widetilde{\delta}$ can be graded with respect to ν as:

$$\left[\nu, \widetilde{\delta} \right] = \sum_{m,n \geq 0, \; m+n \geq 1} (m+n) \, \widetilde{\delta}(m+n) \; , \quad \text{(A.4)}$$

In general the Spectral Sequence method insures that the $\widetilde{\delta}$ cohomology is isomorphic to the solutions $\widetilde{\Delta}(z,\bar{z})$ of the system:

$$\begin{cases} \widetilde{\delta}(m+n)\widetilde{\Delta}(z,\bar{z}) = 0 \\ \widetilde{\delta}^\dagger(m+n)\widetilde{\Delta}(z,\bar{z}) = 0 \end{cases} . \quad \text{(A.5)}$$

or (in other words) $\widetilde{\Delta}(z,\bar{z})$ are zero modes of the Laplacians $\left\{ \widetilde{\delta}(m+n), \widetilde{\delta}^\dagger(m+n) \right\}$, such that:

$$\left\{ \widetilde{\delta}(m+n), \widetilde{\delta}^\dagger(m+n) \right\} \widetilde{\Delta}(z,\bar{z}) = 0 \quad \text{(A.6)}$$

The first level of filtration will select the part of the operator which does not contain any Φ, Π fields:

$$\widetilde{\delta}(0) = \sum_{m,n \geq 0} \left\{ \left(\partial^m \bar{\partial}^n \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \phi_{0,0}(z,\bar{z})}(z,\bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \gamma_{1,1}(z,\bar{z})} \right. \right.$$

$$\left. + \partial^m \bar{\partial}^n \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \mu(z,\bar{z})} \frac{D}{D\partial^m \bar{\partial}^n \eta(z,\bar{z})} + \partial^m \bar{\partial}^n \frac{\delta \mathbf{\Gamma}_0^{Cl}}{\delta \bar{\mu}(z,\bar{z})}(z,\bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\eta}(z,\bar{z})} \right) \quad \text{(A.7)}$$

$$+ \partial^m \bar{\partial}^n \left((-\gamma_{1,1}(z,\bar{z})\partial \phi_{0,0}(z,\bar{z}) - \eta(z,\bar{z})\partial \mu(z,\bar{z}) - \partial(\mu(z,\bar{z})\eta(z,\bar{z})) - \bar{\partial}\eta(z,\bar{z}) \right.$$

$$\left. - \bar{\eta}(z,\bar{z})\partial\bar{\mu}(z,\bar{z}) - \partial(\mu^2(z,\bar{z})\bar{\eta}(z,\bar{z})) \right) \frac{D}{D\partial^m \bar{\partial}^n \zeta(z,\bar{z})}$$

$$+ \partial^m \bar{\partial}^n \left((-\gamma_{1,1}(z,\bar{z})\bar{\partial} \phi_{0,0}(z,\bar{z}) - \bar{\eta}(z,\bar{z})\bar{\partial}\bar{\mu}(z,\bar{z}) - \bar{\partial}(\bar{\mu}(z,\bar{z})\bar{\eta}(z,\bar{z})) - \partial\bar{\eta}(z,\bar{z}) \right.$$

$$-\eta(z,\bar{z})\bar{\partial}\mu(z,\bar{z}) - \bar{\partial}(\bar{\mu}^2(z,\bar{z})\eta(z,\bar{z}))\Big) \frac{D}{D\partial^m\bar{\partial}^n\bar{\zeta}(z,\bar{z})}$$

$$-\partial^m\bar{\partial}^n(\bar{\partial}\rho^{z^n}(z,\bar{z}) - \mu\partial\rho^{z^n}(z,\bar{z}) + n\partial\mu\rho^{z^n}(z,\bar{z})) \frac{D}{D\partial^m\bar{\partial}^n\beta_{\bar{z}}^{z^n}(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n(\partial\bar{\rho}^{\bar{z}^n}(z,\bar{z}) - \bar{\mu}\bar{\partial}\bar{\rho}^{\bar{z}^n}(z,\bar{z}) + n\bar{\partial}\bar{\mu}\bar{\rho}^{\bar{z}^n}(z,\bar{z})) \frac{D}{D\partial^m\bar{\partial}^n\bar{\beta}_z^{\bar{z}^n}(z,\bar{z})}\Big)$$

The previous operator being nilpotent, we filter it with the counting operator of the fields :

$$\widetilde{\delta}(0)_0 = \sum_{m,n\geq 0}\Big(\partial^m\bar{\partial}^n\Big(-2c_0(\partial\bar{\partial})\phi_{00}\Big) \frac{D}{D\partial^m\bar{\partial}^n\gamma_{1,1}(z,\bar{z})} \tag{A.8}$$

$$+\partial^m\bar{\partial}^n\Big(-\bar{\partial}\eta(z,\bar{z})\Big) \frac{D}{D\partial^m\bar{\partial}^n\zeta(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n\Big(-\partial\bar{\eta}(z,\bar{z})\Big) \frac{D}{D\partial^m\bar{\partial}^n\bar{\zeta}(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n(\bar{\partial}\rho^{z^n}(z,\bar{z})) \frac{D}{D\partial^m\bar{\partial}^n\beta_{\bar{z}}^{z^n}(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n(\partial\bar{\rho}^{\bar{z}^n}(z,\bar{z})) \frac{D}{D\partial^m\bar{\partial}^n\bar{\beta}_z^{\bar{z}^n}(z,\bar{z})}\Big)$$

$$\tag{A.9}$$

So we can calculate the Laplacian:

$$\left\{\widetilde{\delta}(0)_0, \widetilde{\delta}(0)_0^\dagger\right\} \tag{A.10}$$

getting:

$$\left\{\widetilde{\delta}(0)_0, \widetilde{\delta}(0)_0^\dagger\right\} = \sum_{m,n\geq 0}\Big(\partial^m\bar{\partial}^n\Big(2c_0(\partial\bar{\partial})\phi_{00}(z,\bar{z})\Big) \frac{D}{D\partial^m\bar{\partial}^n 2c_0(\partial\bar{\partial})\phi_{00}(z,\bar{z})} \tag{A.11}$$

$$+\partial^m\bar{\partial}^n\gamma_{1,1}\frac{D}{D\partial^m\bar{\partial}^n\gamma_{1,1}(z,\bar{z})} + \partial^m\bar{\partial}^n\Big(\bar{\partial}\eta(z,\bar{z})\Big)\frac{D}{D\partial^m\bar{\partial}^n\bar{\partial}\eta(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n\zeta\frac{D}{D\partial^m\bar{\partial}^n\zeta(z,\bar{z})} + \partial^m\bar{\partial}^n\Big(\partial\bar{\eta}(z,\bar{z})\Big)\frac{D}{D\partial^m\bar{\partial}^n\partial\bar{\eta}(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n\bar{\zeta}\frac{D}{D\partial^m\bar{\partial}^n\bar{\zeta}(z,\bar{z})} + \partial^m\bar{\partial}^n(\bar{\partial}\rho^{z^n}(z,\bar{z}))\frac{D}{D\partial^m\bar{\partial}^n(\bar{\partial}\rho^{z^n})(z,\bar{z})}$$

$$+\partial^m\bar{\partial}^n\beta_{\bar{z}}^{z^n}(z,\bar{z}))\frac{D}{D\partial^m\bar{\partial}^n\beta_{\bar{z}}^{z^n}(z,\bar{z})}$$

$$+\partial^m \bar{\partial}^n (\partial \bar{\rho}^{\bar{z}^n}(z,\bar{z})) \frac{D}{D\partial^m \bar{\partial}^n (\partial \bar{\rho}^{\bar{z}^n})(z,\bar{z})} + \partial^m \bar{\partial}^n \bar{\beta}_z^{\bar{z}^n}(z,\bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\beta}_z^{\bar{z}^n}(z,\bar{z})} \Bigg)$$

$$(A.12)$$

The solution of the system (A.5) will be into the kernel of the previous operator, so we can decompose (A.12) into a sum of positive terms of moduli (with respect of our definition of adjoint); and each of them will be identically zero.

It is matter of calculation to derive that the cohomological space does not depend on

$$\beta_z^{z^n}(z,\bar{z}), \beta_{\bar{z}}^{z^n}\bar{(z,\bar{z})}, \gamma_{1,1}(z,\bar{z}), \zeta(z,\bar{z}), \bar{\zeta}(z,\bar{z})$$

,

$$\partial \bar{\partial} \phi_{00}(z,\bar{z})$$

$$\bar{\partial} \rho^n(z,\bar{z}), \partial \bar{\rho}^n(z,\bar{z})$$

and their derivatives.

We remark that the $\rho^{z^n}(z,\bar{z})$ dependence of (A.8) will come from the inhomogeneous term of the β BRS transformation, that is we have to require: $(\bar{\partial} \rho^{z^n}(z,\bar{z}) - \mu \partial \rho^{z^n}(z,\bar{z}) + n\partial \mu \rho^{z^n}(z,\bar{z})) \neq 0$.

The next filtration will provide

$$\widetilde{\delta}(0)_1 = \sum_{m,n \geq 0} \Bigg(\partial^m \bar{\partial}^n \Big(-c_0(-\bar{\mu}\bar{\partial} - \bar{\partial}\bar{\mu})(\bar{\partial})\phi_{00} - \partial(\mu\partial)\phi_{00} \Big) \frac{D}{D\partial^m \bar{\partial}^n \gamma_{1,1}(z,\bar{z})} \qquad (A.13)$$

$$\partial^m \bar{\partial}^n \Big(-c_0(\partial \phi_{00} \partial \phi_{00}) \Big) \frac{D}{D\partial^m \bar{\partial}^n \eta(z,\bar{z})}$$

$$\partial^m \bar{\partial}^n \Big(-c_0(\bar{\partial}\phi_{00} \bar{\partial}\phi_{00}) \Big) \frac{D}{D\partial^m \bar{\partial}^n \bar{\eta}(z,\bar{z})}$$

$$+\partial^m \bar{\partial}^n \Big(-\eta(z,\bar{z})\partial\mu(z,\bar{z}) - \partial(\mu(z,\bar{z})\eta(z,\bar{z}))$$

$$-\bar{\eta}(z,\bar{z})\partial\bar{\mu}(z,\bar{z}) \Big) \frac{D}{D\partial^m \bar{\partial}^n \zeta(z,\bar{z})}$$

$$+\partial^m \bar{\partial}^n \Big(-\bar{\eta}(z,\bar{z})\bar{\partial}\bar{\mu}(z,\bar{z}) - \bar{\partial}(\bar{\mu}(z,\bar{z})\bar{\eta}(z,\bar{z}))$$

$$-\eta(z,\bar{z})\bar{\partial}\mu(z,\bar{z})) \Big) \frac{D}{D\partial^m \bar{\partial}^n \bar{\zeta}(z,\bar{z})}$$

$$-\partial^m \bar{\partial}^n (-\mu\partial\rho^{z^n}(z,\bar{z}) + n\partial\mu\rho^{z^n}(z,\bar{z})) \frac{D}{D\partial^m \bar{\partial}^n \beta_{\bar{z}}^{z^n}(z,\bar{z})}$$

$$+\partial^m \bar{\partial}^n (-\bar{\mu}\bar{\partial}\bar{\rho}^{\bar{z}^n}(z,\bar{z}) + n\bar{\partial}\bar{\mu}\bar{\rho}^{\bar{z}^n}(z,\bar{z})) \frac{D}{D\partial^m \bar{\partial}^n \bar{\beta}_z^{\bar{z}^n}(z,\bar{z})} \Bigg)$$

whose Laplacian zero modes, in the space of those of (A.12) will eliminate, after standard calculations, the dependence of the cohomology space from $\eta, \bar{\eta}$, their derivatives, and from $(\partial \partial)\phi_{00}(z, \bar{z})$, $(\bar{\partial}\bar{\partial})\phi_{00}(z, \bar{z})$, $\rho^n, \bar{\rho}^n, n \neq 0$ and their derivatives,

We have so shown that the cohomology space does not depend on the negative charged external fields, except the terms depending from underivated $\rho^n, \bar{\rho}^n, n = 0$

Having so eliminated the $\Phi.\Pi.$ negative charged fields content, the B.R.S. operator (A.3) strictly recall the one discussed in [1]; so the next calculations will be similar as the ones in this references.

The second filtration will led to the operator:

$$\tilde{\delta}(1) \equiv \partial c(z, \bar{z})S + \partial \bar{c}(z, \bar{z})T + \bar{\partial}c(z, \bar{z})\bar{T} + \bar{\partial}\bar{c}(z, \bar{z})\bar{S} + R(1) . \tag{A.14}$$

where:

$$S^0 = \sum_{m,n \geq 0} \left\{ m \left(\partial^m \bar{\partial}^n \phi_{0,0}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \phi_{0,0}(z, \bar{z})} + \partial^m \bar{\partial}^n \mu(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \mu(z, \bar{z})} \right. \right.$$

$$+ \partial^m \bar{\partial}^n \bar{\mu}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\mu}(z, \bar{z})} + \partial^m \bar{\partial}^n \lambda(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \lambda(z, \bar{z})} + \partial^m \bar{\partial}^n \bar{\lambda}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\lambda}(z, \bar{z})} \right)$$

$$+ \partial^m \bar{\partial}^n \lambda(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \lambda(z, \bar{z})}$$

$$\left. - \partial^m \bar{\partial}^n \mu(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \mu(z, \bar{z})} + \partial^m \bar{\partial}^n \bar{\mu}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{\mu}(z, \bar{z})} \right\} ,$$

$$S^1 = \sum_{m+n \geq 1} \left\{ m \left(\partial^m \bar{\partial}^n c(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n c(z, \bar{z})} + \partial^m \bar{\partial}^n \bar{c}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \bar{c}(z, \bar{z})} \right) \right.$$

$$\left. - \partial^m \bar{\partial}^n c(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n c(z, \bar{z})} \right\} .$$

Note that S is nothing else that the "little z" indices counting operator $N_z(\downarrow) - N_z(\uparrow)$; that is:

$$S = N_\partial + N_\lambda + N_{\bar{\mu}} - N_\mu - N_c \equiv N_z(\downarrow) - N_z(\uparrow) \tag{A.15}$$

Moreover $S^\dagger = S$; similarly:

$$\bar{S} = N_{\bar{\partial}} + N_{\bar{\lambda}} + N_\mu - N_{\bar{\mu}} - N_{\bar{c}} \equiv N_{\bar{z}}(\downarrow) - N_{\bar{z}}(\uparrow) , \tag{A.16}$$

and $\bar{S}^\dagger = \bar{S}$.

Furthermore:

$$T^0 = \sum_{m+n \geq 1} \left\{ m \left(\partial^{m-1} \bar{\partial}^{n+1} \phi_{0,0}(z, \bar{z}) \frac{D}{D\partial^m \bar{\partial}^n \phi_{0,0}(z, \bar{z})} \right. \right.$$

$$+\partial^{m-1}\bar{\partial}^{n+1}\mu(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\mu(z,\bar{z})} + \partial^{m-1}\bar{\partial}^{n+1}\bar{\mu}(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\bar{\mu}(z,\bar{z})}$$

$$+\partial^{m-1}\bar{\partial}^{n+1}\lambda(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\lambda(z,\bar{z})} + \partial^{m-1}\bar{\partial}^{n+1}\bar{\lambda}(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\bar{\lambda}(z,\bar{z})}\Bigg)$$

$$+\partial^{m}\bar{\partial}^{n}(\lambda\mu)(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\lambda(z,\bar{z})}$$

$$-\partial^{m}\bar{\partial}^{n}\mu^{2}(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\mu(z,\bar{z})}\Bigg\} + \frac{D}{D\bar{\mu}(z,\bar{z})} \ ,$$

and

$$T^{1} = \sum_{m+n\geq 2}\Bigg\{ m\left(\partial^{m-1}\bar{\partial}^{n+1}c(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}c(z,\bar{z})} + \partial^{m-1}\bar{\partial}^{n+1}\bar{c}(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\bar{c}(z,\bar{z})}\right)$$

$$-\partial^{m}\bar{\partial}^{n}c(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\bar{c}(z,\bar{z})}\Bigg\} + \bar{\partial}c\frac{D}{D\partial c(z,\bar{z})} \equiv T_{2}^{1} + \bar{\partial}c\frac{D}{D\partial c(z,\bar{z})} \ .$$

$$R(1) = \sum_{k+l\geq 2}\left(\partial^{k}\bar{\partial}^{l}c(z,\bar{z})R_{kl} + \partial^{k}\bar{\partial}^{l}\bar{c}(z,\bar{z})\bar{R}_{kl}(z,\bar{z})\right) \ , \tag{A.17}$$

$$R_{kl}(z,\bar{z}) = \sum_{m+n>k+l;m\geq k;n\geq l}\begin{pmatrix} m \\ k \end{pmatrix}\begin{pmatrix} n \\ l \end{pmatrix}\times$$

$$\left(\partial^{m-k+1}\bar{\partial}^{n-l}c(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}c(z,\bar{z})} + \partial^{m-k+1}\bar{\partial}^{n-l}\bar{c}(z,\bar{z})\frac{D}{D\partial^{m}\bar{\partial}^{n}\bar{c}(z,\bar{z})}\right) \ ,$$

and $\bar{R}_{kl} = R_{k+1,l-1}$.

The Spectral Sequence analysys can be applied to $\widetilde{\delta}(1)$, since:

$$\widetilde{\delta}(1)^{2} = 0 \ . \tag{A.18}$$

Filtering now with:

$$\nu' = 1 + \partial c\frac{D}{D\partial c(z,\bar{z})} + \bar{\partial}\bar{c}\frac{D}{D\bar{\partial}\bar{c}(z,\bar{z})} + 2\left(\bar{\partial}c\frac{D}{D\bar{\partial}c(z,\bar{z})} + \partial\bar{c}\frac{D}{D\partial\bar{c}(z,\bar{z})}\right) \ , \tag{A.19}$$

we have the finite filtration:

$$\left[\nu',\widetilde{\delta}(1)\right] = \sum_{n=1}^{4} n\,\widetilde{\delta}'(n) \ , \tag{A.20}$$

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with

$$
\begin{aligned}
\widetilde{\delta}'(1) &= R(1) \\
\widetilde{\delta}'(2) &= \partial c(z,\bar{z})S + \bar{\partial}\bar{c}(z,\bar{z})\bar{S} \\
\widetilde{\delta}'(3) &= \partial\bar{c}(z,\bar{z})T_2^1 + \bar{\partial}c(z,\bar{z})\bar{T}_2^1 \\
\widetilde{\delta}'(4) &= \partial\bar{c}(z,\bar{z})\bar{\partial}c(z,\bar{z})\left(\frac{D}{D\partial c(z,\bar{z})} - \frac{D}{D\bar{\partial}\bar{c}(z,\bar{z})}\right)
\end{aligned}
\tag{A.21}
$$

$$\tag{A.22}$$

and we have to solve:

$$
\begin{cases}
\widetilde{\delta}'(n)\widetilde{\Delta}(z,\bar{z}) = 0 \\
\widetilde{\delta}'^{\dagger}(n)\widetilde{\Delta}(z,\bar{z}) = 0
\end{cases}
\quad \text{for } n = 1, \ldots, 4.
\tag{A.23}
$$

which is equivalent to:

$$
<\widetilde{\Delta}(z,\bar{z})|\left\{\widetilde{\delta}'^{\dagger}(n), \widetilde{\delta}'('n)\right\}|\widetilde{\Delta}(z,\bar{z})>= ||\widetilde{\delta}(n)\widetilde{\Delta}(z,\bar{z})||^2 + ||\widetilde{\delta}'^{\dagger}(n)\widetilde{\Delta}(z,\bar{z})||^2 = 0 ,
\tag{A.24}
$$

where the scalar product is the one induced through our definition of the adjoint.

First for n=2 we get:

$$
\begin{aligned}
<\widetilde{\Delta}(z,\bar{z})|\left\{\widetilde{\delta}'^{\dagger}(2), \widetilde{\delta}'(2)\right\}|\widetilde{\Delta}(z,\bar{z})> &= ||\widetilde{\delta}'(2)\widetilde{\Delta}(z,\bar{z})||^2 + ||\widetilde{\delta}'^{\dagger}(2)\widetilde{\Delta}(z,\bar{z})||^2 \\
&= ||S\widetilde{\Delta}(z,\bar{z})||^2 + ||\bar{S}\widetilde{\Delta}(z,\bar{z})||^2 = 0 ,
\end{aligned}
\tag{A.25}
$$

which is solved by:

$$
\begin{cases}
S\widetilde{\Delta}(z,\bar{z}) = (N_z(\downarrow) - N_z(\uparrow))\widetilde{\Delta}(z,\bar{z}) = 0 \\
\bar{S}\widetilde{\Delta}(z,\bar{z}) = (N_{\bar{z}}(\downarrow) - N_{\bar{z}}(\uparrow))\widetilde{\Delta}(z,\bar{z}) = 0
\end{cases} ,
\tag{A.26}
$$

where $\widetilde{\Delta}(z,\bar{z})$ do not contain underivated $c(z,\bar{z}), \bar{c}(z,\bar{z})$ ghosts, which are the only fields display an upper "little "index content.

So (A.26) implies that the "little" indices indices have to be saturated. Now this constraint imposed by the cohomology, combined with the fact that underivated ghosts are absent; it is easy to realize that space-time derivatives of $c(z,\bar{z})$ and $\bar{c}(z,\bar{z})$ of order greater than two are absent, since in this case any "little" indices saturation can be insured.

So the n=1 condition:

$$
\begin{cases}
\widetilde{\delta}'(1)\widetilde{\Delta}(z,\bar{z}) = 0 \\
\widetilde{\delta}'^{\dagger}(1)\widetilde{\Delta}(z,\bar{z}) = 0
\end{cases}
$$

is easily verified.

Proceeding further, for n=3 we have:

$$<\widetilde{\Delta}(z,\bar{z})|\left\{\widetilde{\delta}'^{\dagger}(3),\widetilde{\delta}'(3)\right\}|\widetilde{\Delta}(z,\bar{z})> = ||\widetilde{\delta}'(3)\widetilde{\Delta}(z,\bar{z})||^2 + ||\widetilde{\delta}'^{\dagger}(3)\widetilde{\Delta}(z,\bar{z})||^2$$

$$= ||T^0\widetilde{\Delta}(z,\bar{z})||^2 + ||\bar{T}^0\widetilde{\Delta}(z,\bar{z})||^2$$

$$+ <\widetilde{\Delta}(z,\bar{z})|\partial\bar{c}(z,\bar{z})[T,T^{\dagger}]\frac{D}{D\partial\bar{c}(z,\bar{z})}|\widetilde{\Delta}(z,\bar{z})>$$

$$+ <\widetilde{\Delta}(z,\bar{z})|\bar{\partial}c(z,\bar{z})[\bar{T},\bar{T}^{\dagger}]\frac{D}{D\bar{\partial}c(z,\bar{z})}|\widetilde{\Delta}(z,\bar{z})>$$

$$+ <\widetilde{\Delta}(z,\bar{z})|\bar{\partial}c(z,\bar{z})[\bar{T},T^{\dagger}]\frac{D}{D\partial\bar{c}(z,\bar{z})}|\widetilde{\Delta}(z,\bar{z})>$$

$$+ <\widetilde{\Delta}(z,\bar{z})|\partial\bar{c}(z,\bar{z})[T,\bar{T}^{\dagger}]\frac{D}{D\bar{\partial}c(z,\bar{z})}|\widetilde{\Delta}(z,\bar{z})>$$

$$= ||T^0\widetilde{\Delta}(z,\bar{z})||^2 + ||\bar{T}^0\widetilde{\Delta}(z,\bar{z})||^2 + ||\frac{D}{D\bar{\partial}c(z,\bar{z})}\widetilde{\Delta}(z,\bar{z})||^2$$

$$+ ||\frac{D}{D\partial\bar{c}(z,\bar{z})}\widetilde{\Delta}(z,\bar{z})||^2 + ||L(\ldots)\frac{D}{D\bar{\partial}c(z,\bar{z})}\widetilde{\Delta}(z,\bar{z})||^2 + ||M(\ldots)\frac{D}{D\partial\bar{c}(z,\bar{z})}\widetilde{\Delta}(z,\bar{z})||^2 = 0 ,$$

where $L(\ldots)$, $M(\ldots)$ are complicated functions.

The posivity of the metric will imply:

$$\frac{D}{D\bar{\partial}c(z,\bar{z})}\widetilde{\Delta}(z,\bar{z}) = \frac{D}{D\partial\bar{c}(z,\bar{z})}\widetilde{\Delta}(z,\bar{z}) = 0 . \tag{A.27}$$

this result tell us that the cohomology space does not contain $\bar{\partial}c(z,\bar{z})$, $\partial\bar{c}(z,\bar{z})$ monomials.

At the end for n=4 we get:

$$<\widetilde{\Delta}(z,\bar{z})|\left\{\widetilde{\delta}'^{\dagger}(4),\widetilde{\delta}'(4)\right\}|\widetilde{\Delta}(z,\bar{z})> = ||\Big(\frac{D}{D\partial c(z,\bar{z})} - \frac{D}{D\bar{\partial}\bar{c}(z,\bar{z})}\Big)\widetilde{\Delta}(z,\bar{z})||^2$$

$$+ ||\frac{D}{D\bar{\partial}c(z,\bar{z})}\frac{D}{D\partial\bar{c}(z,\bar{z})}\widetilde{\Delta}(z,\bar{z})||^2$$

$$- ||\frac{D}{D\bar{\partial}c(z,\bar{z})}\Big(\frac{D}{D\partial c(z,\bar{z})} - \frac{D}{D\bar{\partial}\bar{c}(z,\bar{z})}\Big)\widetilde{\Delta}(z,\bar{z})||^2$$

$$- ||\frac{D}{D\partial\bar{c}(z,\bar{z})}\Big(\frac{D}{D\partial c(z,\bar{z})} - \frac{D}{D\bar{\partial}\bar{c}(z,\bar{z})}\Big)\widetilde{\Delta}(z,\bar{z})||^2 = 0 .$$

which, with the previous results (A.27) gives the information that

$\widetilde{\Delta}(z, \bar{z})$ does not contain combination of monomials $\partial c(z, \bar{z}) - \bar{\partial}\bar{c}(z, \bar{z})$

Collecting together the results, the Φ, Π charged sector will only contain elements of the type:

$$\widetilde{\Delta}(z, \bar{z}) = \left(\partial c(z, \bar{z}) + \bar{\partial}\bar{c}(z, \bar{z})\right)\Delta_0(z, \bar{z}) , \tag{A.28}$$

But:

$$\partial c(z, \bar{z}) + \bar{\partial}\bar{c}(z, \bar{z}) = \widetilde{\delta}\ln\left(\lambda(z, \bar{z})\bar{\lambda}(z, \bar{z})(1 - \mu\bar{\mu}(z, \bar{z}))\right) . \tag{A.29}$$

So if we enlarge the basis of the cohomology by introducing the non local (in μ!) functions:

$$\{\ln\lambda, \ln\bar{\lambda}\} ,$$

the $\widetilde{\delta}$-cohomology will be empty.

On the other hand in the Φ, Π uncharged space we have to verify:

$$\begin{cases} S^0\Delta_0(z, \bar{z}) = (N_z(\downarrow) - N_z(\uparrow))\Delta_0(z, \bar{z}) = 0 \\ \bar{S}^0\Delta_0(z, \bar{z}) = (N_{\bar{z}}(\downarrow) - N_{\bar{z}}(\uparrow))\Delta_0(z, \bar{z}) = 0 \\ T^0\Delta_0(z, \bar{z}) = 0 \\ \bar{T}^0\Delta_0(z, \bar{z}) = 0 \end{cases} . \tag{A.30}$$

So we have the following theorem:

Theorem *The ghost sectors (Φ-Π charge sectors) of the $\widetilde{\delta}$-cohomology in the space of analytic functions of the fields, where the fields λ and $\bar{\lambda}$ satisfy the equation (1.5), and completed with the terms $\{\ln\lambda, \ln\bar{\lambda}\}$, seen as independent fields depend only on terms containing underivated ρ which multiply zero ghost sector elements of the cohomology.*

The zero ghost sector is, on the other hand, non-trivial only in the part which contains matter fields. Its elements will contain no free z and $\bar{z}$ indices, i.e. they are "scalar-like" quantities with respect to "little indices" but can hold the tensorial content with respect the "big indices" Z and $\bar{Z}$. A generic element of this space will be a $(h, \bar{h})$-conformal quantity of the form

$$[f(\partial_Z^m \partial_{\bar{Z}}^n \phi_{0,0}(Z, \bar{Z}))]_{h,\bar{h}},$$

where f is an analytic function, (polynomial).

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